UMONS
Université de Mons

Faculté Warocqué
d'Economie et de Gestion

Real Option Value: a practical case applied to solar panel in Belgium

Directeur : Franck VENMANS

Co-Directeur : Matthieu DUVINAGE

Mémoire de fin d'études présenté par

Rémi VALEPIN

en vue de l'obtention du diplôme

- de Master en sciences de gestion, à finalité spécialisée

Année académique 2018-2019

Acknowledgements

Thanks to Franck Venmans and Matthieu Duvinage for their help and assistance during this research and to the collaborators of Belfius Bank&Insurance which advised me on some practical aspects.

Table of Contents

Summary of the research

Much research has focused on the evaluation of real options in the context of energy investment projects, it is mainly in this sector that the uncertainties are the largest in estimating the costs and benefits generated by an investment.

The real options approach makes possible to take all these uncertainties into account and therefore the assessment of the case of solar panels in Belgium is very relevant in this respect. These have been the subject of numerous legislative changes both as regards the granting of aid or subsidies than the various related taxes. These changes have created many uncertainties for potential investors who at times have generated installation races.

The different real option valuation models used (analytical solutions and simulations using a Least-Square Monte-Carlo method) converge towards a solution whose value is positive and means an optimal choice by waiting indefinitely. Comparing with the return of a stock exchange investment, solar panel could produce higher profitability with lower risks. An estimation of the return gives a result about 11% in Walloon region, 12% in Flemish region and 28%/year in Brussels region. With this level of profits, invest now is a good alternative as it provides a protection against the growing electricity price in the country (which is interesting for low-incomes people) and legislative changes. Other advantages come from the responsible investment behavior as it helps to contribute to the energy transition towards a more ecological and sustainable world and provide stable cash-flows along the installation life.

Keyword: Solar panel; Real Option Value; Investment; Energy; Least-Square Monte Carlo

Résumé de la recherche

De nombreuses recherches se sont concentrées sur l'évaluation d'options réelles dans le cadre de projet d'investissement en matière énergétique, c'est principalement dans ce secteur que les incertitudes sont les plus grandes quant à l'estimation des coûts et bénéfices générés par un investissement.

L'approche par les options réelles permet justement de prendre toutes ces incertitudes en compte et c'est pourquoi l'évaluation du cas des panneaux solaires en Belgique est dans ce sens très intéressant. Ceux-ci ont fait l'objet de nombreux changements législatifs tant en ce qui concerne les octrois d'aides ou de subsides que les différentes taxes y afférentes. Ces modifications ont entrainé de nombreuses incertitudes pour les potentiels investisseurs générant par moment des courses à l'installation.

Les différents modèles d'évaluation d'options réelles utilisés (solutions analytiques et simulations via une méthode de Least-Square Monte-Carlo) convergent vers une solution dont la valeur est positive et signifie qu'un choix optimal serait d'attendre indéfiniment. Comparé au rendement d'un investissement boursier, un panneau solaire pourrait générer une plus grande rentabilité avec moins de risques. Une estimation du rendement donne une rentabilité d'environ 11% en région wallonne, 12% en région flamande et 28%/an en région bruxelloise. Avec ce niveau de profits, investir maintenant est une bonne alternative car cela offre également une protection contre la hausse du prix de l'électricité dans le pays (ce qui est intéressant pour les personnes à faibles revenus) et d'éventuelles modifications législatives. D'autres avantages peuvent provenir du comportement responsable de cet investissement car il contribue à la transition énergétique vers un monde plus écologique et plus durable et fournit en même temps des flux de trésorerie stables tout au long de la vie de l'installation.

Mots clés: Panneaux solaires; Option réelle; Investissement; Energie; Least-Square Monte Carlo

Samenvatting van het onderzoek

Veel onderzoek heeft zich gericht op de evaluatie van de reële opties in het kader van energie-investeringsprojecten, het is vooral in deze sector dat de onzekerheden het grootst zijn als het gaat om het schatten van de kosten en baten van een investering.

De reële optiebenadering maakt het mogelijk om rekening te houden met al deze onzekerheden en daarom is de beoordeling van het geval van de zonnepanelen in België in dit opzicht zeer relevant. Deze zijn het voorwerp geweest van talrijke wetswijzigingen, zowel wat de toekenning van steun of subsidies betreft als de verschillende belastingen die daarmee verband houden. Deze wijzigingen hebben veel onzekerheden gecreëerd voor potentiële investeerders, die soms tot installatiewedstrijden hebben geleid.

De verschillende gebruikte reële optiewaarderingsmodellen (analytische oplossingen en simulaties met behulp van een Least-Square Monte-Carlo methode) convergeren naar een oplossing waarvan de waarde positief is en die een optimale keuze betekent door eindeloos te wachten. In vergelijking met het rendement van een beursinvestering kan een zonnepaneel een hogere rentabiliteit met lagere risico's opleveren. Een schatting van het rendement geeft een resultaat van ongeveer 11% in het Waalse Gewest, 12% in het Vlaamse Gewest en 28%/jaar in het Brusselse Gewest. Met dit winstniveau is investeren nu een goed alternatief omdat het een bescherming biedt tegen de stijgende elektriciteitsprijs in het land (wat interessant is voor mensen met een laag inkomen) en tegen veranderingen in de wetgeving. Andere voordelen vloeien voort uit het investeringsgedrag, aangezien het bijdraagt tot de energietransitie naar een meer ecologische en duurzame wereld en zorgt voor een stabiele cashflow tijdens de levensduur van de installatie.

Trefwoorden: Zonnepanelen; Reële optie; Investeringen; Energie; Least-Square Monte Carlo

I. Introduction

Every day, people need to make choices and for some decisions it can involve a huge amount of money, it's the case with the investment decisions. This research will be focused on an investment decision in renewable energy and more specifically in a solar panel installation in Belgium. With the context of climate change, this kind of behavior becomes more and more important for our societies.

The question is not "Do we need to invest?" but "When do we need to invest?"

It can be very difficult to answer to this query because energy investments are subject to a high uncertainty about the probable costs and benefits, which involved a higher risk and a higher return to compensate this risk-taking position accordingly to the CAPM theory.

Decisions are generally based on multiple models which give an indication on the profitability of the project, it's the case for the Net Present Value (NPV) or the Internal Rate of Return (IRR) method but their major weakness is that they don't incorporate the role of the uncertainty and flexibility involved on every decision process. With those techniques, investors would require an expected return of 25% or a pay-back time of 4 years or less, which is high in comparison with a stock exchange investment.

It's why the theory of real option could help; this technique incorporates those missing variables in the valuation of the investment decisions and contributes to reduce the expected return by a decrease of the underlying risk through a financial option. An application example of this method could be to wait 1 year before investing and avoiding the risk of a legislation change due to an election.

Multiple researches have been made on the subject to value investment decisions in copper mining, nuclear energy or hydraulic energy. Photovoltaic is on its side subject to a smaller literature with a focus on the Asian context. The results tend to show a significant importance of the electricity price and government subsidy regime. The Belgian case is in this sense a perfect test to value this kind of option due to the high electricity price and variations of the government subsidies.

The document will be divided into 2 major parts:

- A theorical part that introduces the main techniques to value investment as NPV, approach of real option and mathematical tools (Brownian motions) to value such financial products
- A practical part that describes the Belgian context of solar panel, a literature review of similar projects and 3 different models to value the real option.

The results will give a value for the option allowing to determine the best time to invest considering all the underlying risk involved on a such project.

Part I :

Theorical Background

II. How to value a decision to invest

1. Main methods

Multiple decision tools are used to value the opportunity of an investment project, the most principal will be presented here on a non-exhaustive basis. Most of the formulas come from the reference book of Myers [1] on corporate finance theory.

a) Net Present Value (NPV)

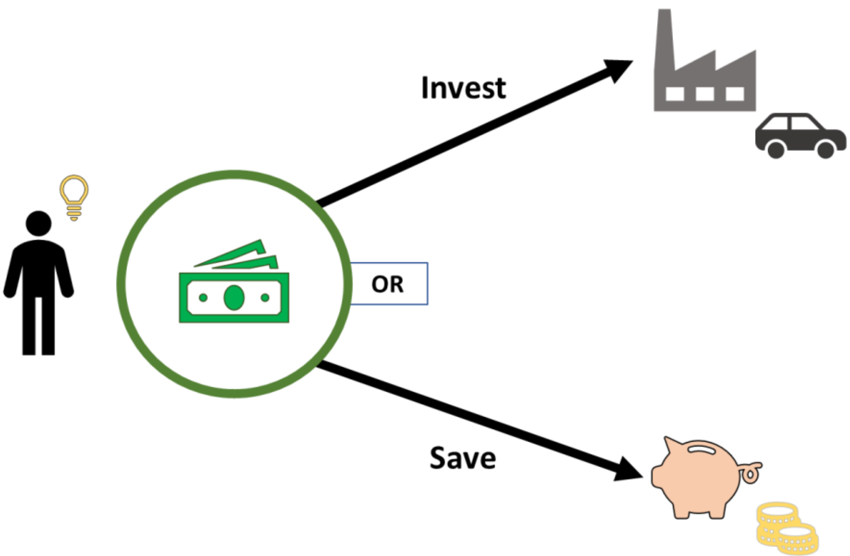
The first method, and the most famous, to value an investment project is the net present value (NPV). This technique requires the expected cash-flow generated by the project, generally represented by an investment cost at the first period and a regular income produced by the exploitation along the life of the project. They are actualized with an interest rate which depends of the perspective and the profile of the investor. Corporate financial theory advices to use a rate that reflects the equity cost of the investor which relies most of the time on the CAPM model. Those flows are actualized at a power corresponding to their distance in time from the actual situation, then all the numbers of the previous operation are summed. This result gives a value that can be positive or negative. If it's positive, the investor will invest in the project. If not, the investor should keep his money and save it.

$$NPV\ (discrete\ time) = \sum_{t=o}^{n} \frac{R_t}{(1+i)^t}$$
$$NPV\ (continuous\ time) = \sum_{t=o}^{n} R_t * e^{i*t} \quad (1)$$

Reference: Myers, page 101

Figure 1 - Investment decision

An economic agent has the choice between to invest in a risky investment project or save the money at a free risk rate



A practical example will now be introduced with standard variables, it will be used through the theorical part to understand all the different concepts developed here and their implications on the profitability of an investment project.

Example 1: Basic situation

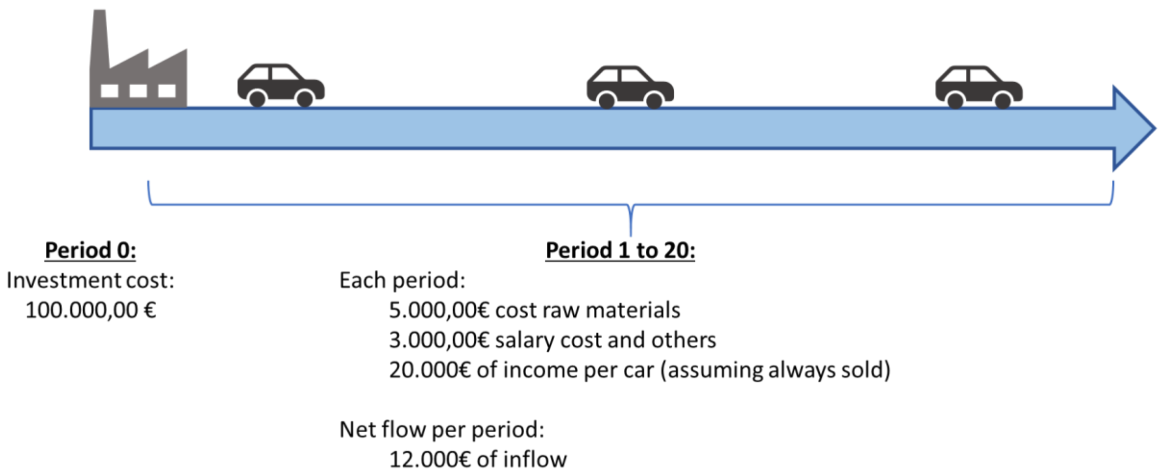
A business executive wants to invest in a machine to increase the production of the firm. Suppose that the factory produces only one product, a car based on a diesel motorization. If the project seems profitable, he will invest. If not, he will save his money. The interest rate on the saved money is close to 0%, the investor will invest only if the project is profitable

Based on an analysis of the project, the following costs and incomes are expected:

- 100.000,00 € for the investment cost at period 0
- 5.000,00€ per period for the cost of raw materials to produce the car
- 3.000,00€ per period for the salary, the marketing and administrative costs
- The machine produces 1 car per period and each car is sold at 10.000€
- The machine produces cars for 20 periods

Figure 2 - Basic situation

The basic situation is based on an investment in a machine that produces car for 20 years.



NPV valuation

Based on Reuters for Volkswagen, the beta is about 1,47[1], the automobile sector is riskier than the average market because the beta is bigger than 1. We assume that the company as the same risk profile than Volkswagen. Consider a risk-free rate of 2% and a return for the market of 5%.

Variable	Value
Beta	1,47
Risk-free rate	2%
Return of the market	5%

CAPM formula [1, p. 193] give us, the required rate of return for the company:

$$Return_{company} = Risk_{free} + \beta * \left(\mathrm{Return}_{\mathrm{market}} - Risk_{free}\right) \tag{2}$$

Applying (2) with the example, it gives:

2% + 1.47 * (5%-2%) = 6,41%

6,41% is the expected return of the company for the investor, this is the actualization rate that should be used.

First, costs and incomes should be actualized:

- The investment cost (100.000,00€) is in period 0, it's not actualized
- The net inflow (12.000,00€) occurs at each period, the VA function of excel gives the actualized value: 133.172,59€ = VA (r = 6,41% ; n = 20; VC = -12000)

The NPV is: 133.172,59€ - 100.000,00€ = 33.172.59€
As the value is positive, the investor should invest.

b) Internal Rate of Return (IRR)

The internal rate of return is similar to the NPV. Again, it's based on the actualization of the cash-flows, but the interest rate is not a constant. Its value is determined to obtain an NPV of 0. If the interest from this calculation is bigger than the cost of equity, the investor should invest. If it's less, the investor should keep his money.

$$IRR = \sum_{t=o}^{T} \frac{F_t}{(1+i)^t} - C_0, where\ NPV = 0$$

$$F_t represents\ the\ net\ cash\ inflow\ at\ period\ t \tag{3}$$
$$C_0 represents\ the\ total\ initial\ investment\ costs$$

Reference: Myers, page 113

[1] https://www.reuters.com/finance/stocks/overview/VOWG_p.DE, consulted on 13/02/2019
The value is not accurate with the reality and is only assumed for the purpose of the example

IRR valuation

Based on the same data than the NPV valuation, variables have the following values:

Variable	Value
Beta	1,47
Risk-free rate	2%
Return of the market	5%
Return of the company (CAPM)	6,41%

This gives an NPV of 33.172.59€. As the IRR should be the rate of an NPV equals to 0, it should be bigger than 6,41%. After incremental testing and interpolation, the IRR's value is 10,3156% for an NPV of 0 which is coherent with the above formula. IRR's rate is greater than the equity cost, the investor should invest.

c) Pay-back time

This technique is different of the last 2 ones, the goal is to determine after how many years or months the cumulated incomes of the project will be greater than the cumulated outcomes. If the recovery time of the costs based on the incomes is not too large, the investor should invest. The major flaw of the method is that it's not based on theorical justification, some people will do actualization of the flows and some not. The decision will mainly depend of the behavior of the investor.

For logical reasons, some cases appear where nobody would like to invest:

- The pay-back time never happens (negative profitability)
- The pay-back time is greater than the investor's horizon

Outside those cases, not everybody will agree on a pay-back time of 10 years for a defined project. The impatient ones would like to have a recovery time of 5 years and a very long-term horizon of investment could find 20 years as a correct value, even if the project is not profitable and has an NPV below 0 due to the actualization.

Pay-back time valuation

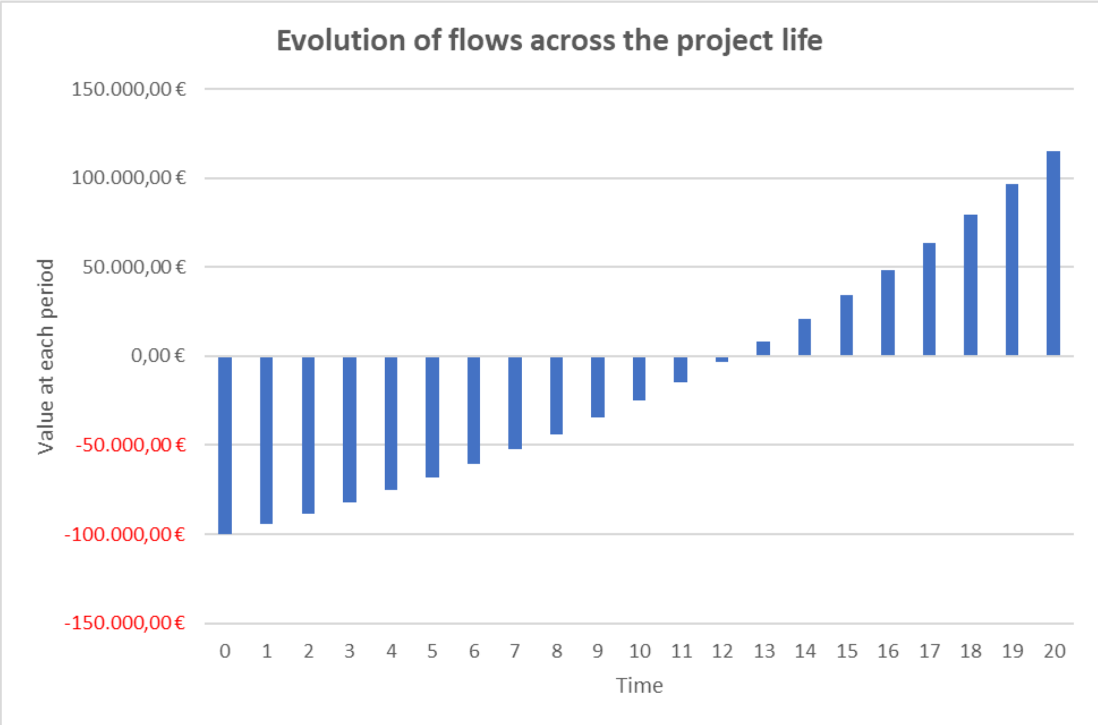
Based on the same data than the NPV and IRR valuation, variables have the following values:

Variable	Value
Beta	1,47
Risk-free rate	2%
Return of the market	5%
Return of the company (CAPM)	6,41%
IRR	10,32%

Computing the value (at each period) of inflows and outflows of the project along the time, we obtain a pay-back time between the period 12 and 13 for a project with a life of 20 years. The break-even point is when the value crosses the negative and positive area. Based on this criterion, a short-term investor (less than 5 years) wouldn't like to invest and a long-term investor (greater than 10 years) should invest, even if the NPV or IRR valuation give a green light to the project.

Figure 3 - Pay-back time technique

The pay-back time of the basic situation is at the 13th year (x-axis is expressed in yearly format) where the line of cumulated cash-flows crosses the value of 0



d) Simple rule of thumb and basic decisions drivers

Some people use very basic technique to decide if they invest or not:

- Cumulated incomes – cumulated outcomes (without actualization):
  People with low financial education can use it in order to have an idea if the project seems profitable

- Multiple of the costs:
  Another possibility is to use a rule of thumb such as an optimal ratio (or multiple). It provides an information on when somebody should invest in a project. This value can be determined by an historic of past transactions or habits. For example, an optimal ratio of 1.5 means that an investor should invest when the expectancy of the incomes generated by the project are 1,5 times bigger that the expected costs. It can be a useful alternative to the previous techniques for people who don't like the time-consuming calculus.

Simple valuation

An investment cost of 100.000,00€ and inflows of 12.000,00€ for 20 periods.
The non-actualized value is 100.000,00€ - (20 * 12.000,00€) = 140.000,00€
As it's greater than 0, he should invest

With a multiple of 1,6 and an investment cost of 100.000,00€, the expected incomes must be greater than 1,6 x 100.000,00€ = 160.000,00€.
As the actualized incomes are about 133.172,59€ (from the NPV valuation), he shouldn't invest.

Summary of the results of the method:

Method	Results
NPV	33.172,59 €
IRR	10,32%
Pay-back	12,5 years
Simple method	140.000,00 €
Multiple ratio	Don't invest

2. Approach with Real Option

a) Objective of the method

In most investment projects, uncertainty is not correctly considered. It will be demonstrated by the 2 following examples. They are commonly valued without uncertainty or just by basic scenarios where all goes well or all goes bad (best case and worst case), but some events are unpredictable and can deter the project's profitability.

The real option valuation is a solution to this problem. The main principle is that a project has many additional options varying with the position of the company or by the information collected with the development of the project. Those options can have various forms (defer, time-to-built, scale, abandon, switch, growth) and will lead to higher NPV, we will discuss of them more precisely in the point 3.b.

Using again the basic situation of chapter 1, the business executive should invest when the NPV is greater than 0, but he needs to consider the inherent risks of the project. Those risks will be represented by many scenarios and events that can impact the investment profitability. Some of them and their associated effects will be demonstrated through examples in the following points.

1. Scandal

In the example situation, the business executive wants to invest in a machine that produces cars with diesel motorization. If for some reasons a diesel scandal appears, customers will lose confidence in this type of technology (ecological reasons, fear of higher taxes, they have deceived by the industry) and the sales will rapidly decrease as the company is active in this business. It leads to a negative impact on the profitability and he will suffer of losses if the decrease of sales is large enough.

[2]

Impact of a scandal

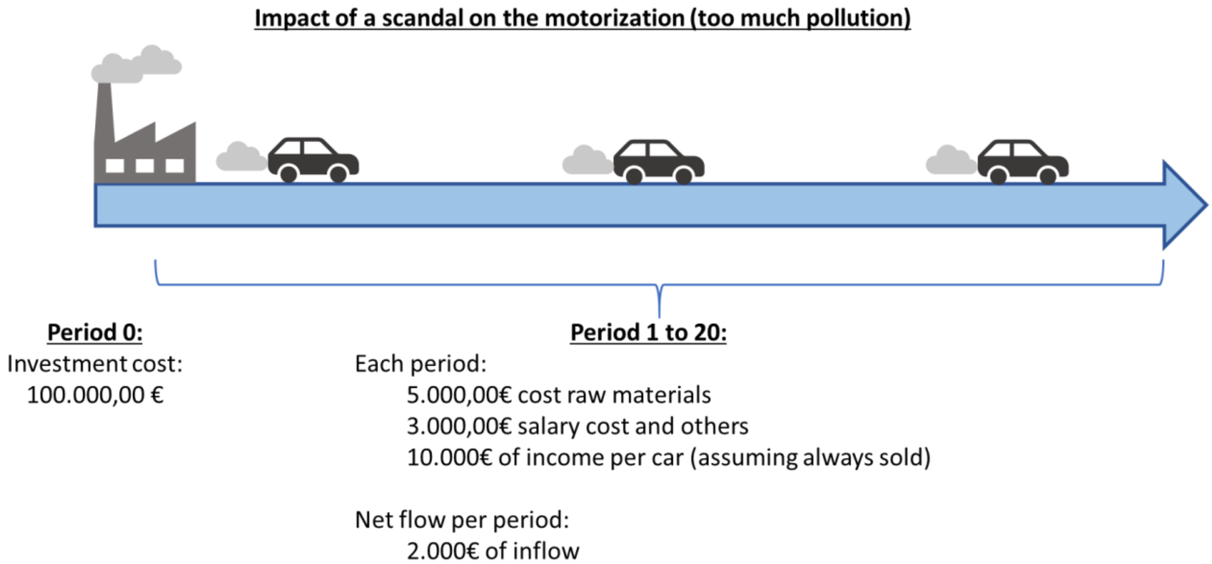
Because the customers loss confidence and expect bigger taxes on this type of motorization, the company needs to set the car price at 10.000,00€ instead of 20.000,00€ to be able to sell the production.
Applying the valuation method, the results give:

Method	Results (Basic)	Results (Scandal)
NPV	33.172,59 €	-77.804,57 €
IRR	10,32%	Close to 0% (no profitability)
Pay-back	12,5 years	Never happen
Simple method	140.000,00 €	-60.000,00 €

On an ex-post basis, the project seems to be deep out the money and he should not have invested (based on the 4 methods).

Figure 4 - Impact of a scandal

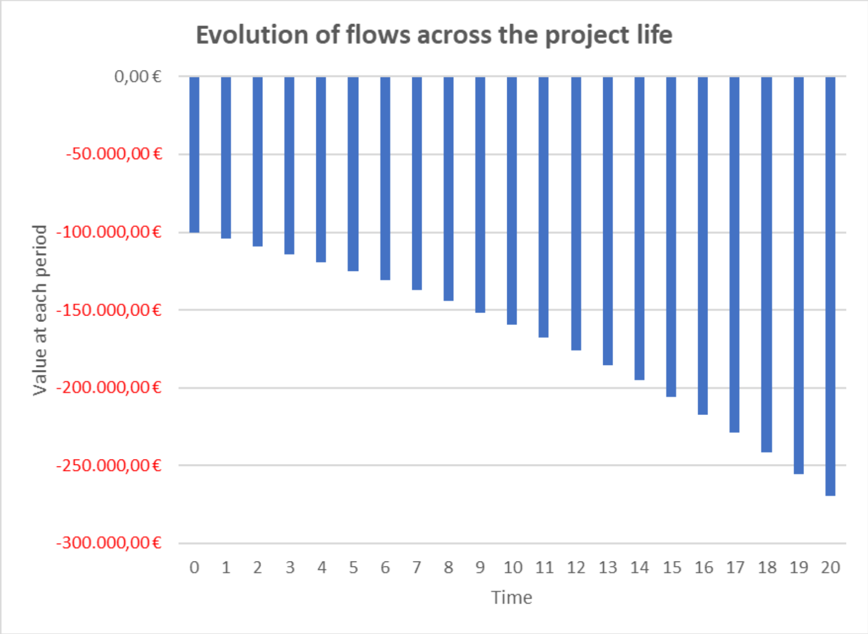
With the impact of a scandal, net flow per period is reduced from 12.000€ to 2.000€. It will have an impact on the profitability of the investment even if the situation seemed profit making at the period 0



[2] Image from https://www.lobservateur-automobiles.com/2018/10/16/dieselgate-audi-condamne-la-facture-sallonge-pour-volkswagen, consulted on 15/06/2019

Figure 5 - Pay-back time with a scandal event

With the impact of the scandal, the pay-back time never happens because the project will never be profitable



2. <u>Skyrocket increase of raw materials needed to construct the car</u>

Assuming that a car is mainly composed of steel in order to resist to many shocks during the vehicle life, the price of this material will be the cost driver of the production and will clearly influence the decision to invest.

If accidentally a trade war starts between USA and China (main producers of steel) and the European and local supplier of steel closes due to economic or social reasons, the price of the steel will skyrocket. It will be difficult to be supplied in raw materials. Thanks to a good manager of the buying department, it will be possible to be supplied in steel by an African supplier but with a high cost (2 times the common price). The price will maintain this level for 3 years.

The machine can produce with a high operational cost which will be passed on the final price of the car. Those vehicles will rapidly become unsellable due to an uncompetitive price, all the other competitors continue to sell cheap cars as they have negotiated many strategic contracts with suppliers around the world with whom they have long-term relationship. It protected them from changes in steel price for the 3 coming years and it was not possible for the investor to negotiate such conditions

Again, both costs and incomes suffered of this unexpected price change. Costs rise due to the trade war and incomes cannot rise due to the strong competition on the market. The investment doesn't bring profitability even the NPV was below 0 at the initial decision to invest.

Impact of a trade war

The event has the net following effects:

- From period 1 to 3, net inflow is equal to 7.000,00€ instead of 12.000,00€

Applying the valuation method, the results give:

Method	Results (Basic)	Results (Trade war)
NPV	33.172,59 €	-22.315,99 €
IRR	10,32%	3,445% < 6,41% (not invest)
Pay-back	12,5 years	15 years
Simple method	140.000,00 €	125.000,00 €

On an ex-post basis, the investment doesn't seem to be profitable with the NPV and IRR criterions. With the Pay-Back time method, a long-term investor can accept to spend money in the project. With the simple method it also seems profitable. The conclusion is more mitigated, even the financial theory would say to save instead of investing.

Figure 6 - Impact of a trade war

A trade war could produce a temporary influence of the net flow which are decreased to 7.000€ for the 3 first period. Again, it's impact negatively the profitability compared to the initial decision

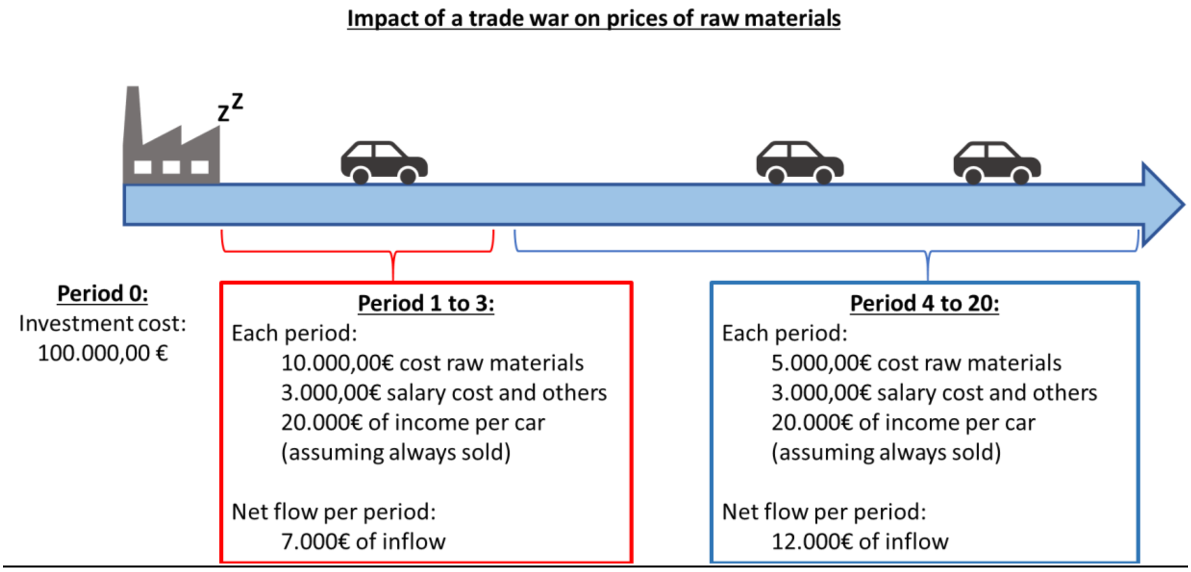
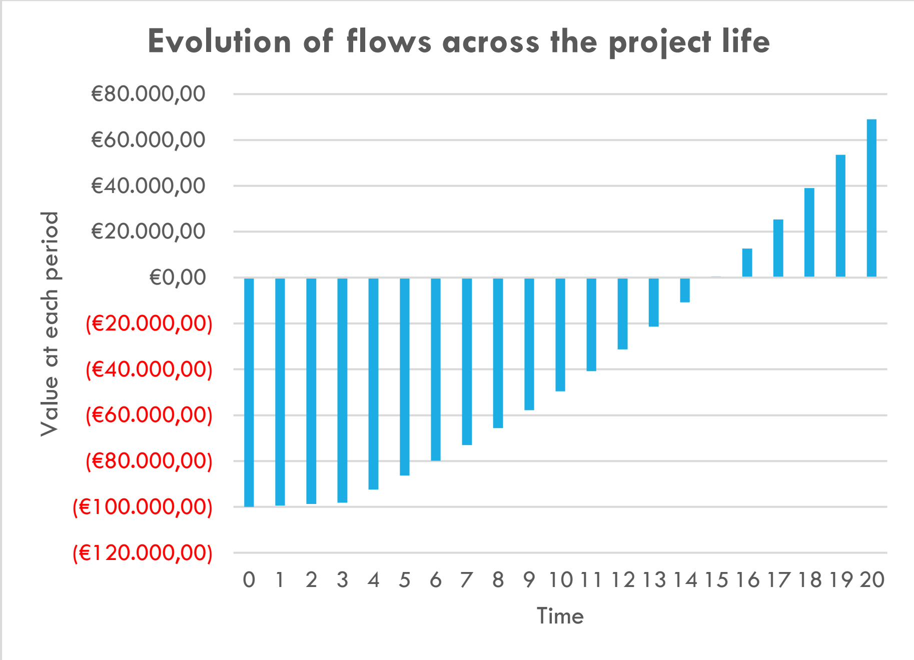


Figure 7 – Pay-back time with a trade war

The pay-back time in this case happens on the 15th years instead of 13th with the basis situation



3. How is it possible to mitigate those risks and events?

The events presented above are exceptional and not predictable, the business executive as the other competitors could not know it in advance. Their effects are more predictable once it happens, a rising operational cost generally leads to less profits and a scandal is not a signal of confidence for the customers.

One of the solutions is the Real Option Valuation (ROV), this technique allows to consider many uncertainty variables. It will now be explained through the following example where the investor can simulate the profitability of the project across the time.

First, he needs to consider the possible variation of steel price (main raw material as explained previously) based on historical data's available on internet[3]. The analysis can consider a very large number of situations, even the trade war scenario. Historical analysis can give information's about previous financial crisis and their sizes. The investor can imagine the impact of some events and by fixing a low rate of occurrence if the data is not available. A similar process can be done with the incomes with the historic of previous sales of the company or of the sector.

Example of events [4]:

A drop of 30% in sales for each financial crisis, every 10 years on average
A drop of 50% in sales for each scandal, every 5 years on average
An increase of 120% for the price of steel, every 8 years on average

[3] https://tradingeconomics.com/commodity/steel provided free data on steel price
[4] Numbers are not based on a real situation, it's only for the logic of the example

What about the investment cost?

The investment cost will certainly have variations in a real-world situation. One solution is to consider a constant investment cost, but it induces problems in case of exceptional increase. A better choice is to consider a variable investment cost with the possibility to increase the size of the investment if it's profitable: an incremental investment. The flexibility granted by the use of real options makes it possible to take decisions adapted to each type of situation or events.

Incremental investment logic

The business executive can have a real option at the beginning of his idea, just a concept of the project. It can be valued by a compound option.

1) He has the right to enter in the calculation of project profitability and associated cash-flows, the cost to enter in this option can be of 250€, the price of an experienced consultant (the automated process allows a low price).
   If, in his mind, the project seems realizable, he activates the option and spends 250€ for the consultant. The report gives him an idea of the potential incomes and outcomes of the project.

2) After it, he can enter in a second option that gives him the right to invest in a small machine with a low production rate but much cheaper than the first project. The cost is around 5.000,00 €. If the second option is worth activating, he buys the small machine.

3) The third option part gives him 3 choices:
   A. Continue to invest in a larger capacity if the project appears profitable.

   B. Pause the production if the price of steel become too high or if the sales are not sufficient. He only loses just some money compared to the first choice and can restart the production if the price goes down enough that it becomes optimal to restart production.

   C. Stop the project if it appears that's unprofitable (abandon). The option is always present, but its value is deeply out the money and will surely never be activated.

   At each time, he has the option to growth, to defer or to stop to limit the losses.

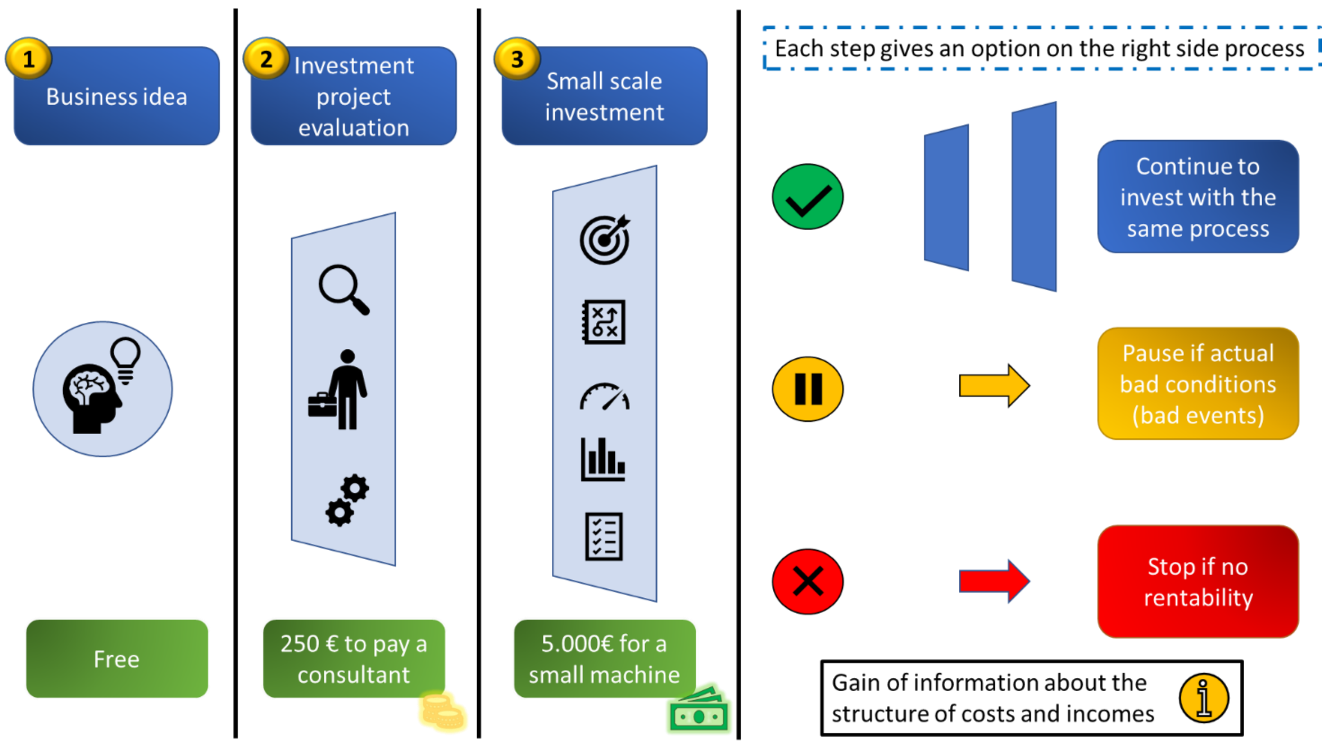
The NPV is converted into an extended NPV (ENPV) where the value is given by the formula [2, p. 355]:

	ENPV = normal NPV + Real options	(4)

The main contribution of the real option theory is to increase the net present value with a greater the flexibility induced by the option. A traditional and negative NPV can become positive into an ENPV. The investment rule is to invest if the ENPV is positive.

Figure 8 - Incremental investment with real option flexibility

To avoid negative events, one possibility is to invest incrementally which grants multiple real options that will increase the profitability and limiting the risks of the investment. At each time, the economic agent has the opportunity to stop the project if it will never be profitable; wait if a bad event happens; continue to invest if it's profitable.



b) Types of options

Many kinds of real option exist, they have been summarized by A. G. &. J. v. d. Bergh [3], this paragraph refers directly to it:

- Defer option:
  This means that the company has the opportunity to invest now or wait and acquire more information's on future market conditions in order to avoid bad events or assess them.

- Time-to-built option:
  This option gives to the holder the possibility to abandon the project if market conditions turn unfavorable. This refer to the incremental investment just explained above.

- Alter operating scale option or the option to expand contract, shut down and restart:
  It's a scalable investment allowing to take advantage of different events and of some market situation

- ❖ Abandon option:
  Allows to abandon the project if it turns unprofitable due to bad market conditions.
  The investor avoids losing money by just waiting.

- ❖ Switch option:
  Gives a flexibility to switch one product to the other when the market conditions turn out to be more favorable.

- ❖ Grow option:
  Allows the investor to increase the capacity of production to take a better advantage of future growth opportunities.

c) Pro's and Con's

As every theory and techniques, ROV has advantages and disadvantages [4]:

Advantages:

- It considers the uncertainty in investment projects which is not considered with the usual techniques or not totally.
- It's a dynamic system, the investor can change his behavior with the time or with new information's and not only at the first period as other methods.
- It can create more investment opportunities because it contributes to value more precisely the projects profitability.

Disadvantages:

- Some people state that a high number of real options contribute to an over valuation of the NPV and could be not accurate with the situation. It can be solved by only considering relevant option kinds directly referred to the project (just keep the essential).
- Difficulties to value real option value and modelized it to the problem context, it must be adapted to each situation and project. It can be solved by using specific Monte Carlo simulation which are able to handle complex modelling.

3. Real Option Valuation

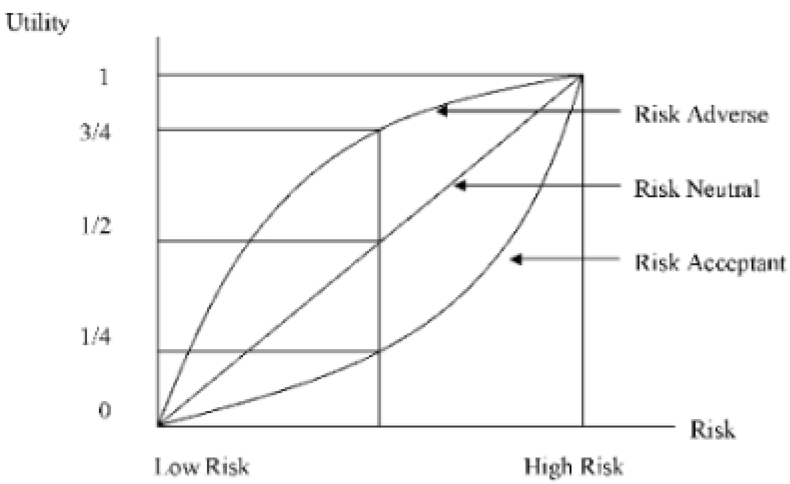
a) Introduction to option valuation

Option valuation is a very large field of research in finance with simple formulas as the Black Scholes model or much complex for the Asian or path dependent option. This research doesn't intend to explain the whole option valuation theory but only the necessary tools to value real option, for further development we refer here to the reference book about this subject [5].

Option valuation is mainly determined on the principle of risk neutral [5, p. 332] Each investor has his own preference in terms of risk, some are more risk adverse and some more risk taker. As it's impossible to know the preference of everybody at every instant, options are valued under risk neutral. The investor will be indifferent to the risk when making an investment decision. This investor is placed in the middle of the risk spectrum as shown on this scheme:

Figure 9 - Risk appetite[5]

Most of the investors are risk adverse and will require higher return (or utility) by unit of risk compared to a risk neutral investor



It allows us to compute the value of the option by non-taking into account the risk preference for the moment. Only the expectancy of the return of the investment will be necessary.

[5] Image from https://netwar.wordpress.com/2007/07/28/marathon-and-risk-preference/

b) Martingale

A martingale is a stochastic process where the expectancy of the future value is based on the available information at time 0, which is its value at time 0. Saying it in a simplified way, the best predictor of a series (a stock) is its present value. It relies directly to the risk-neutral valuation because the expected return (necessary to determine the value of the option) is the present value that is available in most of the cases by looking the spot price of a stock for example.

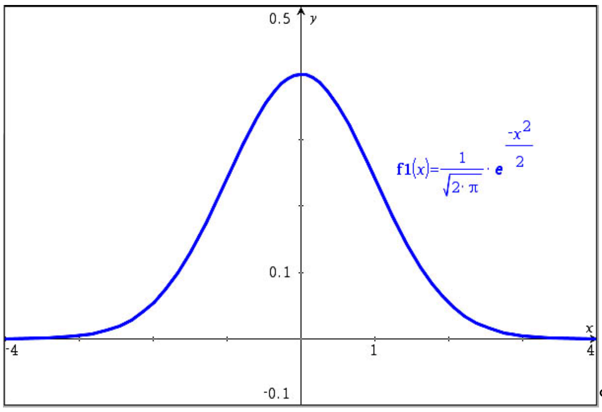
c) Brownian motions

To achieve a martingale, and by the same time a risk neutral valuation, we need to have a random process with a mean of 0 because the process expectancy is the present value (no change on average is assumed in the future). Many distributions tend to a normal distribution when a high number of simulations are used which is the basis of the central limit theorem (CLT)[6]. A logical solution is a random process distributed according to a normal distribution. It's the purpose of a Brownian motion (also called Wiener motion).

Brownian motion is represented by a normal distribution of the event in a portfolio. Some days we will have positive variations that increase the value of a stock, and another day a negative impact. On the long term, those effect are compensated because Brownian motions follow a normal distribution of mean equal to 0 and variance of 1. Those increments are depended of time at the rate $\sqrt{t}$. It's generally presented as follow:

[7]
$$dW = \sqrt{t} * rnorm(0\,,1) \tag{5}$$

It's means that an error or a change will be accumulated at rate 1 by time.



[9]

[6] [25]
[7] Another symbol (dz) is used too in the literature and has the same signification
[9] Image from http://www.maxicours.com/se/fiche/7/1/415671.html, consulted on 16/02/2019

d) Risk neutral valuation

By combining the 2 previous concepts, we can have a free arbitrage rule. Martingale is achieved through Brownian motions; the price of the stock cannot differ from the actual value in a risk neutral perspective. If it was not the case, one can short the stock to obtain a free risk profit if the price is overvalued compared the risk neutral measure or take a long position if the price is undervalued.

To summarize, we need:

1. A stochastic process following a normal distribution (mean 0, variance 1): Brownian motion
2. It gives a martingale, a process where the expectancy is the present value
3. Which gives a free arbitrage rule and a risk neutral valuation

With those concepts, it's possible to compute the risk neutral probability that will be imputed into a formula giving a risk neutral value for the option.

How to consider the risk preference?

It's not useful to have a pricing with risk preference, most of the options are valued in risk neutral probabilities. The changes from one risk world to another (risk-neutral) has been demonstrated through the Girsanov theorem [6, p. 70] and Radon-Nikodym theorem. Those theorems are outside of the scope of this paper and it's not necessary to explain it in more details. The results are now expressed in terms of risk-neutral measure.

4. Mathematical background

To solve an option valuation problem, 4 ways are possible:

- Binomial tree or Decision tree base model [5, p. 272]:
  It consists in a tree where the price of stock can increase with a up probability (p) or decrease with a down probability (d). Some trees include a probability where the price stay at the same level, it's named trinomial tree.
  A solution can be obtained if the final payoff rule at the maturity is known, it allows to compute the backward values in time using the probability rules of the problem which gives a solution at time 0.

- Analytical solution [7, p. 41]:
  The most complex method, it requires to solve the problem by using partial derivative equations (PDE). The result is the most precise compared to the methods, but it doesn't allow to find a solution for complex problems. This method will be explained in detail in the next point.

- Finite Difference Method [5, p. 477]:
  This method is based on the differential equation of the problem (PDE) which is converted into a set of differential equations solved iteratively.

- Monte Carlo Simulation [5, p. 468]:
  It's based on a simulation where the parameters are defined at the initial time of the problem and multiple scenarios are generated using random process. When the number of simulations is large enough, values will converge to the solution of the problem. It requires a lot of time of computation but is able to manage complex problem efficiently.

a) Analytical solution

To find an analytical solution of an option X(t), it's necessary to find the expectancy at time 0 of a stock with spot price S with a strike K, actualized continuously at the rate µ and at the optimal time t'. The solution requires to solve an equation with the following form:

$$X(t) = E_0[e^{-\mu * t'} * (S_t - K)] \qquad (6)$$

As it involves expectation, it cannot be solved directly. The first step is to set parameters of the process:

S can follow a Brownian Motion process:

$$dS = \alpha * dt + \sigma * dW \quad where\ dW = \sqrt{t} * rnorm(0;1) \qquad (7)$$

Alpha (α) represents the drift of the process and Sigma (σ) is the volatility of S

A stock generally follows a Geometric Brownian Motion (GBM), it allows to have a log-normal distribution of the price changes [7, p. 35]:

$$dS = \alpha * S * dt + \sigma * S * dW \quad where\ dW = \sqrt{t} * rnorm(0;1) \qquad (8)$$

For some commodities, a mean-reverting process is used. Stock price tends to go to the mean level ($\bar{S}$) at long term with the rate of reversion (η), it's defined as:

$$dS = \eta * (\bar{S} - S) * dt + \sigma * dW \quad where\ dW = \sqrt{t} * rnorm(0;1) \qquad (9)$$

The next step must be done through a formula: Ito's Lemma.

b) Ito's lemma

Ito's lemma is a Taylor series that defines the derivative of a stochastic process. It requires to write the Taylor series of the main variables as the option is mainly affected by S and by the time t, they will be used in the series. F represents here the value of the option

Write the first derivative of the variables, then the second derivatives, then the higher order, …

$$d\,F(S,t) = \frac{\partial S}{\partial t}dt + \frac{\partial F}{\partial S}dS + \frac{1}{2} * \left[\frac{\partial^2 F}{\partial^2 S}dS^2 + \frac{\partial^2 F}{\partial^2 t}dt^2 + \frac{\partial^2 F}{\partial S \partial t}dS\,dt\right] + \cdots \quad (10)$$

dS is already known with the Brownian motion

In the case of a perpetual option, the first derivative by t of (9) can be eliminated. The originality of the Ito's approach is that the term containing "dt^2" (terms of order equal or bigger than 3) tends faster to 0 than the other term (S), so it can be set equal to 0. The term containing "dS dt" is the correlation between S and t, it can also be considered 0. The variance of dS can be calculated based on the Brownian Motion and applying the same rules for dt.

Ito's lemma finally gives:

$$d\,F(S,t) = \frac{\partial S}{\partial t}dt + \frac{\partial F}{\partial S}dS + \frac{1}{2} * \frac{\partial^2 F}{\partial^2 S} * \sigma^2 * S^2 * dt \quad (11)$$

c) Bellman's equation

The variation in the option value (dF) must be equal to the value of the option (F) multiplied at each time by the risk-free rate (r), this relation avoids any arbitrage opportunity (equation 11). Ignoring the derivative following "t" in (10), reducing all the terms and by equating it to the required rate of return "r" of the project and by considering that the stock gives a dividend at the rate (δ) [8, p. 148], the results can be computed in one formula, the Bellman's equation which gives an optimal solution based on the actual value.

$$\delta = \mu - \alpha = r + \phi * \sigma * \rho_{S;M} - \alpha$$

Where:

- δ is a kind of measure of the dividend rate (even if the stock doesn't pay any dividend)
- μ is the risk adjusted trend
- α is the trend of the Brownian Motion
- ϕ is the market price of risk which can define as

$$\phi = \frac{(r_M - r)}{\sigma_M} \quad (12)$$

- r_M is the expected return on the market
- σ_M is the volatility of that return
- σ is the volatility of the Brownian Motion
- $\rho_{S;M}$ is the correlation between the stock (S) and the market (M)

$$\delta = \mu - \alpha = r + \frac{(r_M - r)}{\sigma_M} * \sigma * \rho_{S;M} - \alpha$$

$$F(S) * r * dt = dF \quad (13)$$

$$(r - \delta)\, F' + \frac{1}{2} * F'' * \sigma^2 * S^2 - r * F \quad (14)$$

"The principle of optimality can be articulated as follows. An optimal policy has the property that, no matter what state and initial decision, the remaining decisions must be an optimal policy in relation to the state resulting from the first decision "(Bellman, 1957).

d) Partial Differential Equations (PDE)

To solve the Bellman's equation, a function with a suited form to the problem is needed. Classically, 2 forms exist:

- $A * e^{\beta * S}$ for an arithmetic Brownian motion
- $A * S^{\beta}$ for a geometric Brownian motion

  Where A and β represent a constant

As a geometric Brownian motion have been used, the second form must be used. β has generally 2 roots (the first is positive, the second is negative) and the function takes the form:

$$A_1 * S^{\beta_1} + A_2 * S^{\beta_2} \quad (15)$$

Boundary conditions have to be fixed to simplify the problem [7, p. 49], it depends if the option is a call or a put. As most of the real option are valued as a call, the following boundary conditions are applied:

- If the stock price is equal to 0, the option price will also be 0:
  F (0) = 0

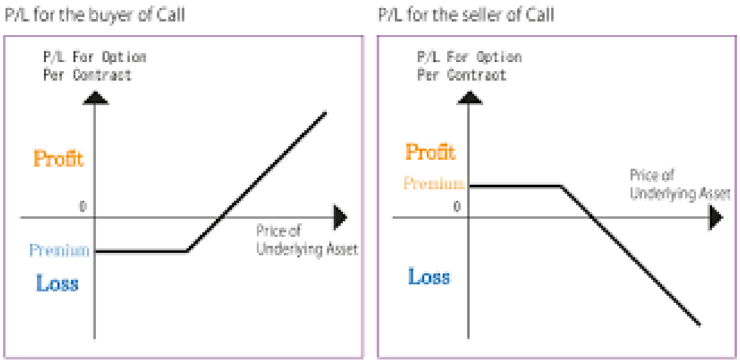
- At the optimal price (S*), the option value must be equal to its payoff:
  F (S*) = S* - K

- Smooth passing condition: the slope of the option value must be equal to the slope of the payoff at the optimal level (S*)
  F'(S*) = 1

Boundary condition	Call	Put
1°	F (0) = 0	$\lim_{S \to \infty} F(S) = 0$
2°	F (S*) = S* - K	F (S*) = K - S*
3°	F'(S*) = 1	F'(S*) = -1
Results	$A_2 = \beta_2 = 0$ Only keep the positive root	$A_1 = \beta_1 = 0$ Only keep the negative root

Figure 10 - Pay-off of a call and a put

A call is a long position on a financial product and a put a short position[10]



e) Solution of the system

Solution to the system is given by the following formula which results of a basic root calculation.

$$A_1 = \frac{S^* - K}{(S^*)^{\beta_1}}$$

$$\beta_1 = 0.5 - \left(\frac{(r-\delta)}{\sigma^2}\right) + \sqrt{\left(\frac{r}{\sigma^2} - 0.5\right)^2 + \frac{2 * r}{\sigma^2}} \tag{16}$$

$$A_2 = \beta_2 = 0$$

$$F^* = \frac{\beta_1}{\beta_1 - 1} * K$$

To obtain the problem solution of a perpetual option, the formula only requires the value of S and the value of the strike K. The value of F* means the optimal value that S needs to reach before the optimal time to activate the option or simpler, the level that S must reach before activating the option in order to compensate the uncertainty involved on the underlying. On the next page, a summary scheme of the procedure of how to resolve a classic real option value problem is provided and will be used to solve a more complex model.

[10] Image from https://www.option-dojo.com/en/le/summary.html, consulted on 27/04/2019

Figure 11 - Summary scheme to solve a PDE

Real Option Valuation: Solving PDE - scheme

Definitions of the basis parameters and Brownian Motions

Set the general form of the problem: Benefit variables – Cost variables

1 Calculate variance and covariance between the parameters

2 Calculate the results of increment terms with a board

Increment terms	dt	dV	dI	dW1	dW2
dt	0	0	0	0	0
dV	0	Var(dV)	Cov(dV;dI)	/	/
dI	0	Cov(dV;dI)	Var(dI)	/	/
dW1	0	/	/	dt	$dt * \rho_{V;I}$
dW2	0	/	/	$dt * \rho_{V;I}$	dt

3 Use Ito's Lemma to obtain the components of the PDE

- Substract terms that goes to 0 with the board
- Substitute others values with the help of the board

Set Risk-free portfolio and arbitrage conditions

- Find delta values
- Replace those values
- Dividing by dt

4 Bellman's equation and boundary conditions

5 Find the most suited form of the solutions

General forms: $A * e^{\beta * V}$ $A * V^{\beta}$

- Substitute solution into Bellman's equation
- Dividing by the global variance of the option

Find the roots and constant $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Solution

Part II :

Practical approach

III. Practical approach with a photovoltaic project

This chapter will be focused on a real option valuation on a photovoltaic project. A presentation of the technical functionalities of a solar will be presented, followed by the Belgian context case. The next point will address the current literature on the subject and 3 valuation models will conclude with the results.

1. How works a solar panel

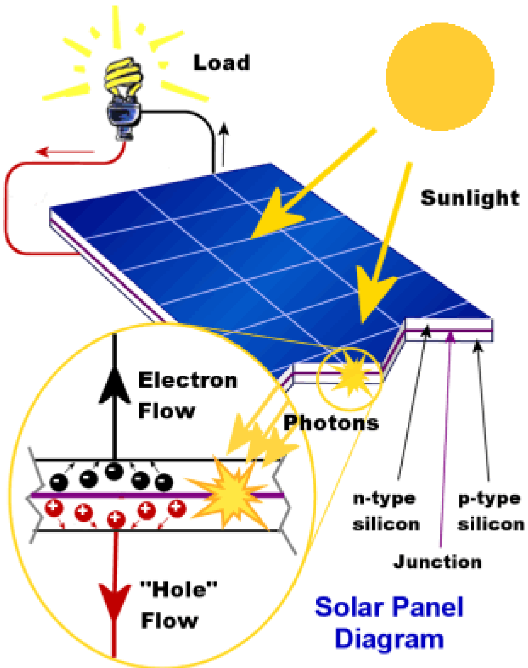
a) Technical functionalities


[11]

How works a solar panel:

Solar panels basically generate electricity from sunlight. Those panels are usually laid on the roof and consist of solar cells: a thin layer of silicon with a negative charge at the top and a positive charge at the bottom protected between two glass plates. With a chemical reaction [9, p. 335], the panels generate direct current which is converted into alternating current by an inverter. The electricity generated goes directly to the electricity system of the house. When more solar power than necessary is produced, the excess goes to the grid and the meter will turn upside down. At night and on dark days, or if the electricity demand is greater than the production, grid will supply the difference to avoid a black-out (need and offer of electricity are always equal).

[11] Image from https://lumiworld.luminus.be/fr/investissements-malins/choisir-vos-panneaux-solaires/



[12]

The nominal power is expressed in Watt peak (called Watt-crête in French – Wc), it corresponds to a unit of measure where:

- It represents the maximal electricity power delivered by the installation
- A standard sunshine of 1000 W/m2 with a heat of 25°C

In 2019, 2 mains technologies are available to construct a solar panel:

- Polycrystalline: the cheapest kind of solar panel but it has a lower production rentability (around 150 Wc/m2)
- Monocrystalline: a bit more expensive than the previous ones but produces more (around 200 Wc/m2)

Productivity is expressed in Wc and the production in Watt-hours (Wh)[13]. The difference comes from the fact that the productivity reflects the production of the installation based on optimal conditions. A solar panel has a life-expectancy about 20 years to 25 years and 10 years for the inverter which must be changed once on the total life of the installation. It also exists a kind of solar panel that convert sunlight into heat, but this research is only focused on the most classical one called "photovoltaic panel".

[12] Image from https://www.pinterest.com/pin/414401603183250304/
[13] Reminder: 1 mWh = 1.000 kWh, useful website about the solar panel energy: http://document.environnement.brussels/opac_css/elecfile/IF%20ENERGIE%20Mod4%20Facteurs%20production%20FR

b) Belgium context

Belgium is a federal state divided into multiple level of decision which have different roles and responsibilities:

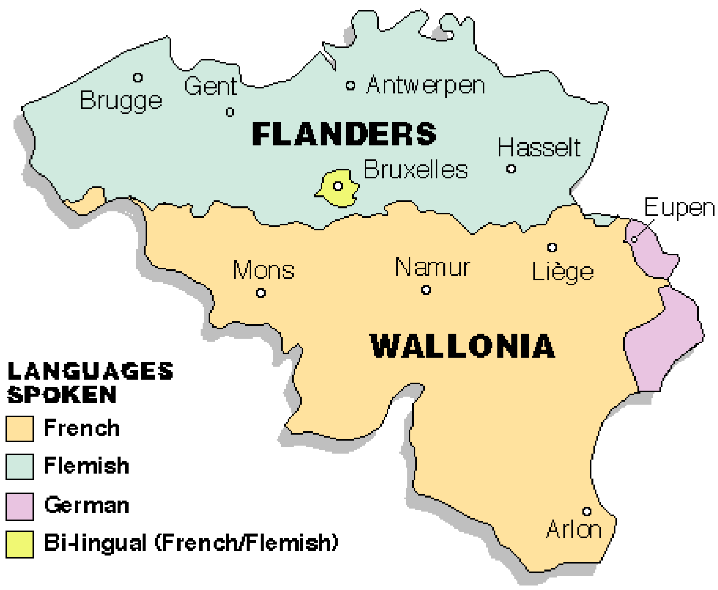
1. Federal level (which is common to the whole country)

2. Region level
   There are 3 regions: Wallonia, Flemish and Brussels

3. Community level
   There are 3 official languages in Belgium and represented by a specific community which concern all the education and people related missions of the state. French community is formed by a major part of Wallonia and of Brussels. Flemish community is formed by the Flemish region and a part of Brussels. German community is formed by a part of Wallonia. This level will not have importance for the research but is provided here for information.

4. City level

   Cities are regrouped in a king of cluster (called "Commune" in French and "Gemeente" in Dutch); this level can grant some extra subsidy or collect specific taxes.



[14]

The following explications concern only solar panel installation with a power inferior to 10 kW, which represents the most widespread size of panel at the household level.

[14] Image from https://chrisnicastro12.wordpress.com/2013/10/01/les-jeunes-et-la-jeunesse/

c) Situation in Walloon region

i. CV system (called Solwatt)

A system of green certificates called in French “Certificats verts” (CV) has been set up under the impulse of the European Union. A directive allowing a new kind of subsidy came into force from the 01/01/2002.

1 CV (for each mWh produced) were granted by installation for 15 years, this amount was determined individually by installation. The applicable regime is defined at the initial decision to invest (when a minimal deposit of 20% have been paid or when a green loan have been contracted).

The 20/12/2007, an energy minister of Wallonia (André Antoine – CDH)[15] wanted to develop the solar energy sector. His main idea was to grant 7 CV instead of 1 CV by mWh produced. The amount of CV decreases slightly with the size of the installation but remains relatively generous. This decision has had the consequence to lead literally to a run to install solar panel on the roofs, this situation was unsustainable for the budget of the region.

The 01/12/2011, the granting period were reduced to 10 years instead of 15 years to decrease the generation of CV, the market were not able to absorb this quantity of CV.

The 01/04/2012, the granting regime were downgraded in the sense that the amount of CV decrease with the life of the installation. On 10 years, an installation receives 60 CV.

The 01/09/2012, the granting regime was again downscaled. On 10 years, an installation receives 50 CV.

The 01/04/2013, a transitory regime has been applied. On average, 1.25 CV were granted for each mWh produced for 10 years.

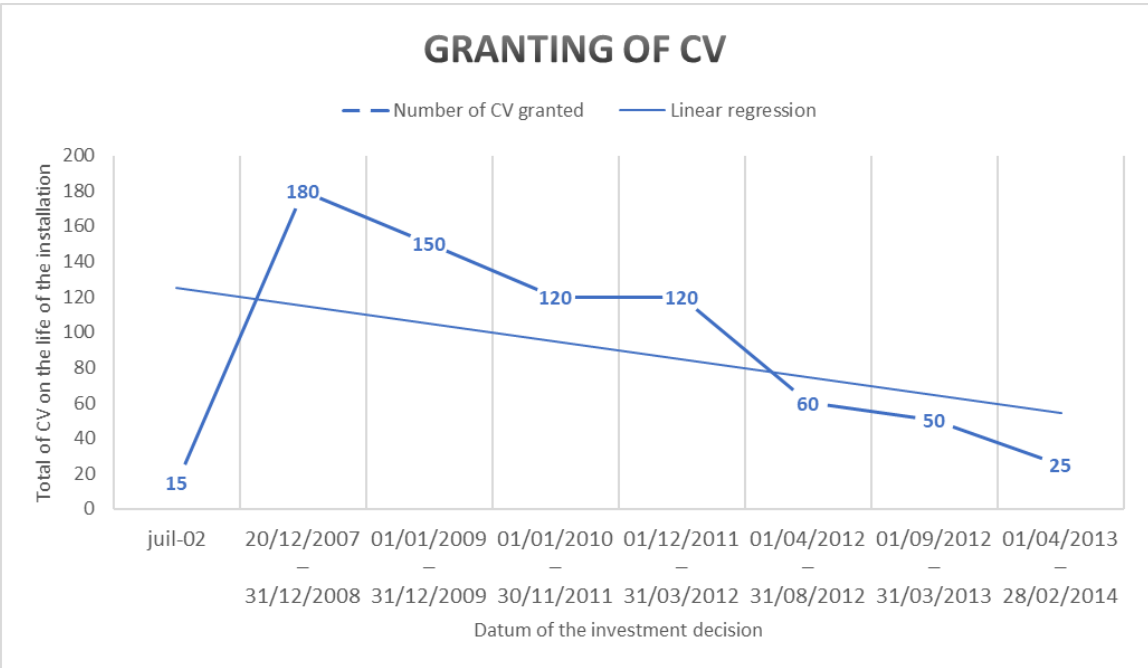
[15] Historic based on multiple press articles:
https://www.rtbf.be/info/belgique/detail_autopsie-episode-6-le-jour-ou-le-photovoltaique-wallon-a-derape?id=9377534
https://www.rtl.be/info/magazine/c-est-pas-tous-les-jours-dimanche/photovoltaique-1061844.aspx
Consulted on 27/04/2019

CV granted by mWh produced for an installation of 10 kw[16]:

Age of the installation	07/2002 – 19/12/2007	20/12/2007 – 31/12/2008	01/01/2009 – 31/12/2009	01/01/2010 – 30/11/2011	01/12/2011 – 31/03/2012	01/04/2012 – 31/08/2012	01/09/2012 – 31/03/2013	01/04/2013 – 28/02/2014
1	1	12	12	12	12	10	8	2,5
2	1	12	12	12	12	9	7	2,5
3	1	12	12	12	12	8	7	2,5
4	1	12	12	12	12	7	6	2,5
5	1	12	12	12	12	6	5	2,5
6	1	12	12	12	12	6	5	2,5
7	1	12	12	12	12	5	4	2,5
8	1	12	12	12	12	4	3	2,5
9	1	12	12	12	12	3	3	2,5
10	1	12	12	12	12	2	2	2,5
11	1	12	12	0				
12	1	12	9	0				
13	1	12	6	0				
14	1	12	3	0				
15	1	12	0	0				
Total	15	180	150	120	120	60	50	25

Figure 12 - Granting regime of CV

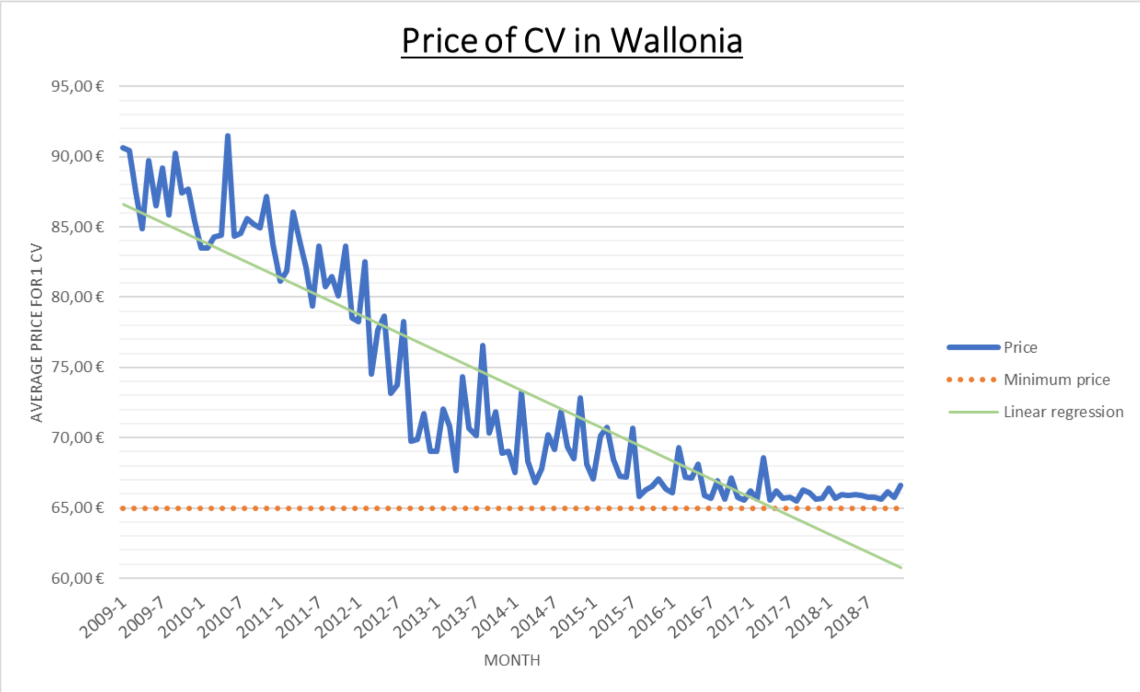
The regime of CV granting has been very generous in 2008 (in a context of financial crisis) and slightly decreases to the end of this system in 2014.



[16] Data based on the report from the CWAPE (Walloon regulator of energy): https://www.cwape.be/?dir=0.5&faqid=124 consulted on 27/04/2019

Figure 13 - Price of CV

Price of CV has followed a decreasing trend to reach the minimum level guarantee by the state at 65€/CV



About the price of a CV, it ranges from a minimum basis (65€)[17] guaranteed by the region (the root of the financial deep) and the amount of the penalty (around 100€)[18]. If values outside of this range happen, an arbitrage opportunity could occur:

- Better to sell the CV at the guarantee price (65€) than the market price
- Pay the penalty (100€) that buying it more expensively on the market

ii. Qualiwatt – temporary direct subsidy

After the crisis of CV, the Walloon region initiates a new formal direct subsidy, more flexible and should avoid the reef of the previous system. Qualiwatt is a direct subsidy (PB) granted annually in cash for 5 years. It was in application from 01/03/2014 to 30/06/2018. The goal is to achieve a **pay-back time of 8 years with a return of 5%.** An extra subsidy (PC) was granted for low-income household to achieve a pay-back time of 8 years with a return of 6,5%. Those values were based on a common installation of 3 kWc. The formula considers multiple variables and becomes more complex.[19]

[17] https://www.cwape.be/?lg=1&dir=3.4.00, consulted on 13/05/2019
[18] https://www.cwape.be/?lg=1&dir=3.4.00, consulted on 13/05/2019
[19] https://www.cwape.be/?dir=6.2.07, consulted on 13/05/2019

The basis prime (PB) for a given period is equal to:

$$PB_i = \min(P; P_{MAX}) * \frac{\frac{SG_j}{5}}{P_{ref}} \quad (17)$$

Where SG is given by

$$SG_j = \Bigg[\left(P_{REF} * I_j * (1 + d_j) \right) + \left(O\&M_j * \sum_{i=1}^{8} (1 + c_{i,j})^i \right) - \Bigg[\sum_{i=1}^{8} \Big[\left(COM_j * (1 + a)^i \right) * \left(ELEC_j * (1 - p_j)^{i-1} \right) + \left(REG_j * (1 + b)^i \right) * \lambda_{i,j} * \left(ELEC_j * (1 - p_j)^{i-1} \right) \Big] \Bigg] \Bigg] \quad (18)$$

- P_{MAX}: 3 kWc
- P_{REF}: 3 kWc
- I: the investment cost
- O&M: Maintenance and operating costs.
  Fixed at 0,75% of the installation cost for each period
  For period 10, an additional amount of 250€ is added to reflect the replacement cost of the undulator
- COM: the price of the commodity
- ELEC: the production of electricity.
  This variable is determined individually for each installation based on the location with the data from European Union research database (available at http://re.jrc.ec.europa.eu/pvgis/) with the following technical specificities: south-east to -south-west orientation, gradient between 15° and 50°, which guarantees 90% of an optimal production
- a: parameter to index the commodity price, fixed at 1%/year
- b: parameter to index the reglementary price, fixed at 3%/year
- c: parameter to index the operating costs to the inflation
  Fixed at 2%/year for all the periods
- d: parameter to index the installation cost
  Fixed at 0%/year for all the periods
- p: parameter to reflect the loss of production due to the age of the installation
  Fixed at 0.5%/year for all the periods
- $\lambda_{i,j}$: Auto consumption assumed for a given period
  Fixed at 100% for period 1 or 2 for most of the periods
  Fixed at 30% for period 2-3 to 20 for most of the periods

The parameters are summarized in the following table, the case of the region of Mouscron (Sibelga/ Ores Mouscron) is provided as example[20]:

Period		I EUR HTVA/kWc	COM EUR TVAC/MWh	REG EUR TVAC/MWh	PB max EUR/year	PC max EUR/year
01/03/2014-30/06/2014	1	2.285,00	95,03	102,20	1.017,00	126,00
01/07/2014-31/12/2014	2	2.285,00	80,45	90,51	1.023,00	133,00
01/01/2015-30/06/2015	3	2.100,00	81,57	88,21	862,00	100,00
01/07/2015-31/12/2015	4	1.900,00	85,66	88,26	708,00	55,00
01/01/2016-30/06/2016	5	1.900,00	92,53	110,57	628,00	5,00
01/07/2016-31/12/2016	6	1.840,00	93,09	109,92	586,00	-
01/01/2017-30/06/2017	7	1.789,00	79,33	112,22	607,00	26,00
01/07/2017-31/12/2017	8	1.725,00	83,87	112,22	544,00	1,00
01/01/2018-30/06/2018	9	1.654,00	86,51	112,26	444,00	-

The last 2 columns represent the maximal value per year of the subsidy in the basis version (most of the cases) and the additional subsidy (only for low-income).

Formula to compute the additional subsidy (PC) is the following:

Set PC to achieve:

$$VAN_j = -I_{TOT,j} + \sum_{i=1}^{20} \frac{CF_{i,j}}{(1+6{,}5\%)^i} = 0 \quad (19)$$

Where

$$CF_{i,j} = [PB_j + PC_j] + [COM_{i,j} * ELEC_{i,j} + REG_{i,j} * \lambda_{i,j} * ELEC_{i,j}] - O\&M_{i,j} \quad (20)$$

PC is finally given by:

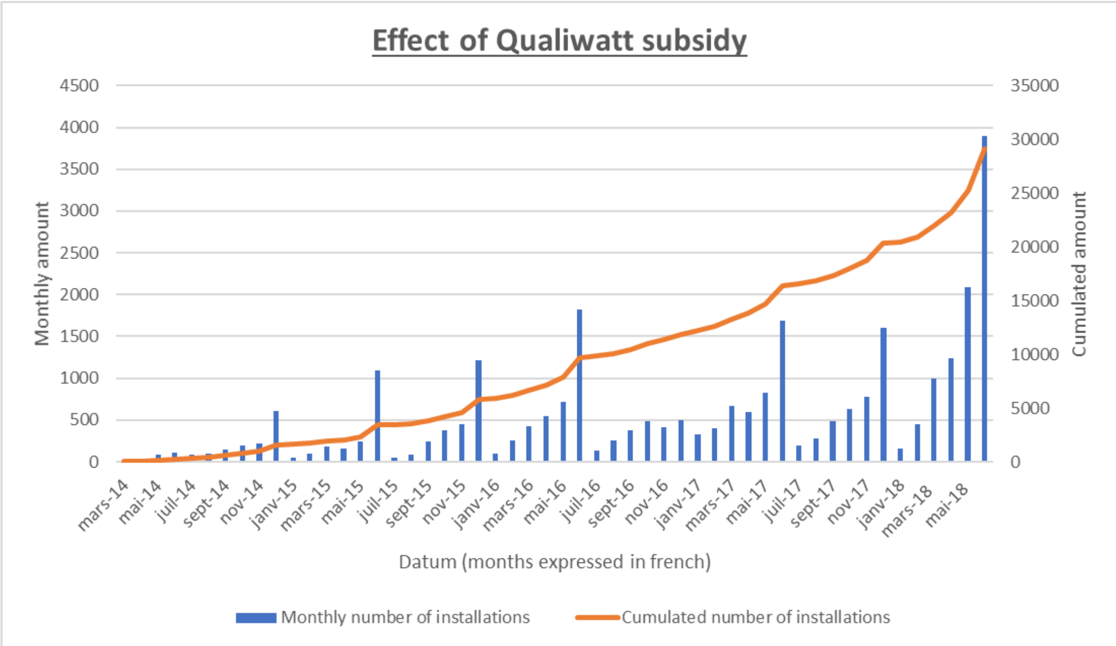
$$PC_j = \min(P; P_{MAX}) * \frac{PC_j}{P_{REF}} \quad (21)$$

The effect of this subsidy was an increase of solar panel installations in Wallonia with peaks occurring just before period switching's as seen below on the graph. A switch has generally the effect to decrease the subsidy, people have an interest to wait the end of the period to know the subsidy amount if they invest the next month. One exception can be noted for the period 7 in January 2017 where the subsidy increases the next period, people don't had interest to invest at that time.

[20] Report of the data's is available at https://www.cwape.be/?lg=1&dir=6.2.09, consulted on 27/04/2019

Figure 14 - Effect of Qualiwatt subsidy

The effect of Qualiwatt subsidy was to increase the number of solar panel installations in Wallonia. People invested especially before a reduction of the subsidy. One exception can be noted for 01/2017 where the subsidy increased at the next period, it was not optimal to invest before.



iii. Uncertainty period and Prosumer tax case

Between 01/07/2018 and 01/01/2020, no new subsidy or legislation change should happen. Solar panel are now providing enough profitability that no subsidy is necessary to obtain a comfortable level of return. To cover the cost of the CV bubble, the region has no other choice to find money through a new tax. The question for the Walloon government is:

Who should pay this tax?

People with solar panel (who benefits of the CV system or Qualiwatt subsidy) or people without solar panel (all the users of the electricity grid)?

Politicians of the Walloon region choose the first option and a new tax should normally be applied on the 01/01/2020. This tax is called “Prosumer tax” and is proportional to the amount of electricity injected in the grid which cause additional costs for the electricity grid manager. An example of the probable amount of the tax is represented in the following table:

Tarif prosumer capacitaire TVAC				
exprimé en €/kWe	2020	2021	2022	2023
AIEG	66,87	67,43	67,27	65,50
AIESH	85,29	86,34	86,50	86,91
ORES NAMUR	87,41	88,16	88,50	88,21
ORES HAINAUT	85,78	85,47	85,95	84,86
ORES EST	98,63	99,39	99,26	98,53
ORES Luxembourg	89,54	90,29	90,63	91,63
ORES VERVIERS	98,84	98,79	99,07	97,08
ORES BRABANT WALLON	78,62	79,24	79,51	79,52
ORES MOUSCRON	78,81	79,67	80,31	82,26
RESA	76,04	77,06	76,87	77,19
REW	89,46	90,75	92,10	88,67

This number represents the tax (in €) for each kWe (minimum between kWc and kVA) of electricity injected in the grid[21], based on statistic auto consumption is valued at 37,76%[22].

Belgian political world is very complex and uncertain, the energy minister of Walloon region (Jean-Luc Crucke – MR) announces in 01/2019 that people who install a solar panel before the 07/2019 will not pay this tax.[23] It induces a new run to invest[24] for the householders whose don't want to pay this additional cost. This kind of behavior is common in Belgium because it's the country with the higher tax pressure in the world[25], people prefer to avoid new taxes if they can[26].

In 03/2019, a new problem occurs: The Walloon government loses his majority[27] and the tax exemption cannot be voted. Nobody knows at this moment if an alternative majority can be found to vote the legislative text. People continues to invest to avoid the probable tax without the certainty if they will pay it.

In 04/2019, a final answer is given: The tax exemption is not granted. Everybody with a solar panel in Wallonia will pay the Prosumer tax, even people whose have invest between the 2 announces have change their behavior and have invested.

The end of the story is not arrived yet, elections will happen at the end of 05/2019 and changes can again happen. Even with the Prosumer tax, the cost of CV debt is too high to be absorbed by the grid manager (Elia). A partial solution have been found with a securitization with a bank (BNP Paribas) which lags the cost on several years and cancels the exceed of CV on the market but a report of the CWAPE tends to show that the electricity price will rise for every consumer at the horizon of year 2025-2030. An increase of electricity price of 61,16€/year can occur for an average household or

---

21 Data from https://www.cwape.be/?dir=0.5&lg=1&faqid=254, consulted on 27/04/2019
22 https://www.cwape.be/?lg=1&dir=7.9, consulted on 27/04/2019
23 https://www.lesoir.be/221290/article/2019-04-29/les-detenteurs-de-panneaux-solaires-devront-finalement-payer-pour-le-reseau
24 One article on the subject: https://www.7sur7.be/7s7/fr/1536/Economie/article/detail/3526143/2019/04/18/Typiquement-belge-la-Wallonie-introduit-le-tarif-prosommateur-et-la-Flandre-le-supprime.dhtml, consulted on 27/04/2019
25 https://www.oecd.org/tax/taxing-wages-belgium.pdf
26 One example can be found in this press article, it's just a global idea of the situation and the behaviors: https://www.rtbf.be/info/societe/detail_la-fraude-fiscale-le-sport-national-quel-est-le-secret-de-la-reussite?id=7975166, consulted on 27/04/2019
27 https://www.lecho.be/economie-politique/belgique/wallonie/le-gouvernement-wallon-a-perdu-sa-majorite/10108306.html, consulted on 27/04/2019

an increase of 17,4751 EUR/mWh [10, p. 93]. For the region, it means a debt of 1.76 billion € to cover the cost of the CV system.

d) Situation in Flemish region

i. CV system (called Groenestroomcertificaten - GSC)

Before the 01/01/2013, one Green Certificates (called GSC[28] in Dutch) and was granted for each 1000 kWh[29] of produced electricity. This CV had a guarantee price of 150€ and was granted for 10 years[30]. The formula is:

$$GSC = ELEC * \frac{1}{1000} \tag{22}$$

Where:

- ELEC is the amount of green electricity produced, expressed in mWh
- GSC is round at the unity (the surplus is transferred to the next month)

From 01/01/2013 to 14/06/2015[31], one CV was granted for a minimum of 1000 mWh of produced electricity + a band factor (Bf). The band factor is like an additional amount of produced electricity to receive 1 GSC, it's reflects the evolution of electricity price.[32] The applicable regime depends of the first time of commissioning.

$$GSC = ELEC * Bf/1000 \tag{23}$$

Where:

- ELEC is the amount of green electricity produced, expressed in mWh
- Bf is the band factor varying through the time

A minimal price of 93€[33] was guaranteed for each GSC for 15 years. This system guarantees a minimal return of 5% annually on the investment.[34]

---

28 https://www.fluvius.be/nl/thema/zonnepanelen/wat-zijn-groenestroomcertificaten, consulted on 27/04/2019
29 https://www.vreg.be/nl/begrippenlijst#groene_stroom, consulted on 27/04/2019
30 https://www.vreg.be/nl/algemene-info-over-steuncertificaten, consulted on 27/04/2019
31 https://www.fluvius.be/nl/thema/zonnepanelen/heb-ik-recht-op-groenestroomcertificaten, consulted on 27/04/2019
32 https://www.energiesparen.be/zonnepanelen-met-een-startdatum-van-1-juli-2014-tot-en-met-31-december-2014, consulted on 27/04/2019
33 https://www.fluvius.be/nl/meer-weten/zonnepanelen/berekening-groenestroomcertificaten
34 https://www.vlaanderen.be/groenestroomcertificaten-voor-zonnepanelen, consulted on 27/04/2019

Band factor (Bf) based on the first date of commissioning[35]

First time of commissioning between	01-01-13	31-12-13	First time of commissioning between	01-01-14	30-06-14
Energy produced between		Bf	Energy produced between		Bf
01-01-13	31-07-13	0,23			
01-08-13	16-02-14	0,28			
17-02-14	31-07-14	0	01-01-14	31-07-14	0,268
01-08-14	26-03-15	0,0847	01-08-14	26-03-15	0,0394
27-03-15	22-08-15	0,818	27-03-15	22-08-15	0,753
23-08-15	31-07-16	0,818	23-08-15	31-07-16	0,753
01-08-16	02-08-17	0,399	01-08-16	02-08-17	0,304
03-08-17	31-07-18	0,163	03-08-17	31-07-18	0,0975
01-08-18	-	0,32	01-08-18	-	0,255

First time of commissioning between	01-07-14	31-12-14	First time of commissioning between	01-01-15	30-06-15
Energy produced between		Bf	Energy produced between		Bf
01-07-14	26-03-15	0			
27-03-15	22-08-15	0,657	01-01-15	22-08-15	0
23-08-15	31-07-16	0,621	23-08-15	31-07-16	0,488
01-08-16	02-08-17	0,162	01-08-16	02-08-17	0,0126
03-08-17	31-07-18	0	03-08-17	31-07-18	0
01-08-18	-	0,133	01-08-18	-	0

Example:

A solar panel installed on the 25/08/2014 has produced 14.000 kWh for the month of 08/2016. The applicable band factor is 0,162. The number of GSC granted is 14.000 * 0,162 / 1000 = 2,268 GSC. For this month, 2 GSC are granted to the installation with a minimal price of 93€

[35] https://www.energiesparen.be/groene-energie-en-wkk/professionelen/monitoring-en-evaluatie/startdatum-na-2013/zonder-brandstofkost/overzicht-bandingfactor-zon, consulted on 27/04/2019

i. Prosumer tax

After the 15/06/2015[36], only installations with a capacity > 10 kWc receive GSC. The subsidy was converted into a tax called "Prosumententarief", the principle is almost the same than the Walloon prosumer tax. It's based on the initial and maximal capacity of the installation (kVA, similar to kWh) and not a tax on the quantity of electricity injected on the grid (like in Wallonia). It's assimilated to a fixed contribution to the grid without direct link to the amount of injected electricity. This system is still in place for 2019 but will be replaced as soon as digital meters will be in place for owners of solar panels[37].

Prosumententax value (€ by kW of kVA) by installation[38]

Operator	01/07/2015-31/07/2015	01.08.2015-31/12/2015	01/01/2016-31/12/2016	01/01/2017-31/12/2017	01/01/2018-31/12/2018	01/01/2019-31/12/2019
Gaselwest	101,63 €	106,32 €	113,03 €	128,15 €	121,46 €	109,24 €
Imea	76,11 €	78,57 €	85,35 €	97,99 €	93,87 €	86,29 €
Imewo	85,08 €	88,44 €	92,89 €	104,88 €	99,61 €	90,15 €
Intergem	82,61 €	85,75 €	80,82 €	93,36 €	87,14 €	77,21 €
Iveka	80,95 €	83,85 €	93,12 €	107,09 €	100,58 €	89,79 €
Iverlek	86,09 €	89,38 €	92,95 €	104,89 €	99,16 €	91,42 €
Gaz sibel	95,78 €	99,06 €	106,94 €	112,71 €	107,67 €	102,49 €
Infrax West	96,18 €	99,76 €	99,74 €	98,69 €	98,98 €	92,83 €
Inter-Energa	96,80 €	99,49 €	95,92 €	89,87 €	91,14 €	85,49 €
Iveg	93,87 €	96,99 €	98,03 €	114,05 €	114,19 €	98,63 €
PBE	94,63 €	97,61 €	93,55 €	96,72 €	94,02 €	92,33 €

Example:
An installation located in Spiere (a small city close to Mouscron) with a maximal capacity of 3 kVA will pay a tax for the year 2016 about 113,03€ * 3 = 339,09€.

[36] https://www.vreg.be/nl/prosumententarief, consulted on 27/04/2019
[37] https://www.vreg.be/nl/begrippenlijst#prosumententarief, consulted on 27/04/2019
[38] https://www.vreg.be/nl/prosumententarief-sector, consulted on 27/04/2019

e) Situation in Brussels region

In Brussels region, a system of CV is applied and still stand for 2019. No other system has been applied, only the methodology of CV granting has changed. 1 CV represents a production of 1 mWh.

Before the 01/07/2011, the number of CV was granted based on the size of the installation[39]:

- 7,27 CV / mWh for the first 20 m²
- 5,45 CV / mWh for the next 40 m²
- 3,63 CV / mWh for the part bigger than 60 m²

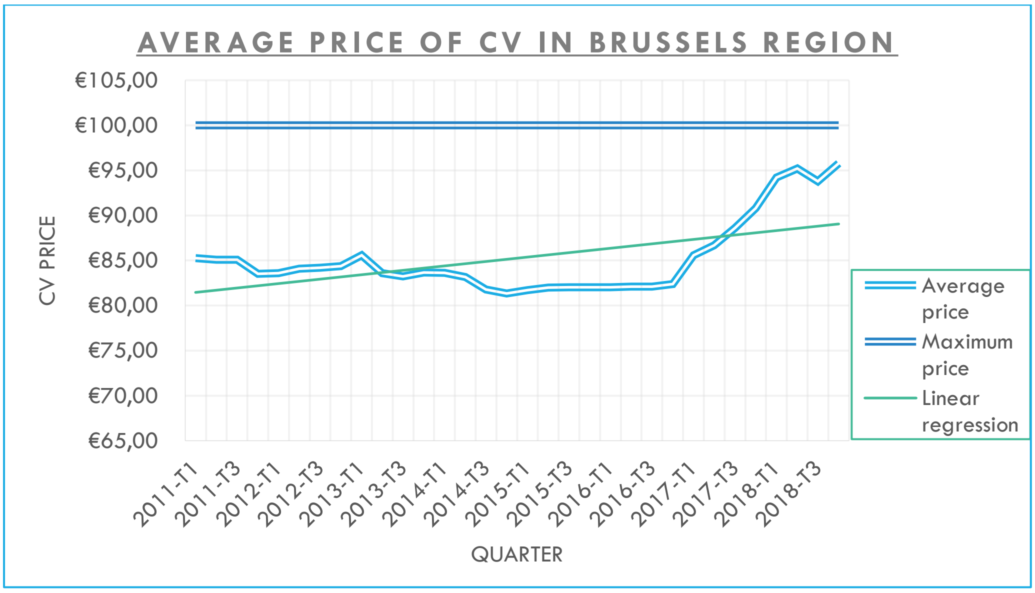
After, the number of CV was based on the power of the installation.[40]

CV granting regime for an installation with a power < 5 kWc

First commissioning datum		CV
01-07-11	20-10-12	7
21-10-12	01-08-13	5
02-08-13	31-01-16	4
01-02-16		3

The CV's are granted for 10 years and have a guarantee price of 65€[41].
The system tends to achieve a pay-back time of 7 years.

Figure 15 - Average price of CV in Brussels region[42]
The average price of CV in Brussels slightly increases, not as the CV in Wallonia



[39] https://environnement.brussels/thematiques/batiment/quest-ce-que-lenergie-verte/certificats-verts/lancien-systeme-de-calcul-pour, consulted on 27/04/2019
[40] https://www.brugel.brussels/acces_rapide/energies-renouvelables-11/mecanisme-des-certificats-verts-35, consulted on 27/04/2019
[41] https://www.brugel.brussels/acces_rapide/energies-renouvelables-11/vendre-les-certificats-verts-38, consulted on 27/04/2019
[42] https://www.brugel.brussels/publication/document/statistiques/2013/fr/observatoire-des-prix-fevrier-a-juin-2013.pdf
https://www.brugel.brussels/publication/document/statistiques/2013/fr/observatoire-des-prix-4e-trimestre-2012-janvier-2013.pdf
https://www.brugel.brussels/publication/document/statistiques/2018/fr/Oservatoire_des_prix_T3.pdf , consulted on 27/04/2019

2. Literature review

Real option is subject to wide area of research, mostly in Strategy and in Energy sector. This method is adapted to deal with the uncertainty involved on energetic investment projects due to the specificity of the energy sector (regulation, monopoly, different equilibrium mechanism to achieve a demand equals to the offer) [11].

K.-T Kim et al. [12, pp. 335-347] have summarized a large part of the literature (see next table) as G. Locatelli [13, pp. 114-131] in a more extensive way.

Summary of real options literature.

Authors	Energy source	Uncertainty	Model	Year	Ref.
Davis et al.	Renewable energy	Electricity annual rate of cost reduction/increase	PDE	2003	[6]
Siddigui et al.	Renewable energy	Fossil fuel price	Tree	2007	[7]
Lee et al.	Renewable energy	Non-renewable energy cost	Tree	2010	[10]
Martinez-Cesena et al.	Hydropower	Electricity price	Tree, Sim	2011	[11]
Fleten et al.	Wind energy	Electricity price	PDE	2004	[13]
Zhou et al.	Wind energy	Electricity price	Sim	2007	[14]
Munoz et al.	Wind energy	Electricity price	Tree, Sim	2009	[15]
Marinez-Cesena et al.	Wind energy	Wind resource	Tree, Sim	2012	[16]
Lee	Wind energy	WTI price	PDE	2011	[17]
Lee et al.	Wind energy	Non-renewable energy price	Tree	2011	[18]
Yoon	Combined heat and power plant	Oil price	Tree	2001	[19]
Lee et al.	Wind energy	Electricity price	PDE	2012	[20]

PDE: partial differential equation and Sim: Simulation.

Most of the researchers used analytical solution or binomial lattice (binomial tree) to solve the problem involved in the real option valuation. It's the case of E. Agliardi et al [14, pp. 1-9] with an analytical valuation for a building renovation, Kyung-Taek et al [12, pp. 335-347] with a lattice model for a R&D model in wind power, S-E Fleten et al [15, pp. 498-506] for a comparison of ROV and NPV in green electricity investment timing. An analytic solution is preferable than a simulation model but when it becomes too complex, the numerical or decision tree become unsolvable (more than 2 stochastic process) with the available techniques.

Monte Carlo simulation instead allows to have more flexibility. It can handle multiple uncertainty and consider their complex interaction. Some assumptions can also be easily changed, whereas it's not possible with analytical solution because it's involved to find a new specific solution to the problem. Normal Monte Carlo requires in the contrary many simulations (more than 10.000 on average) to converge to a stable solution and of course a bigger time of computation. To deal with this weak point, Longstaff & Schwartz [16, pp. 113-147] developed the Least-Square Monte-Carlo to initially value American Option. It was later used for Real Option Valuation, which are essentially American kind option, to approximate the value of the investment option through the time. This model can manage multiple variables and gains in computation time, because it requires less simulations to converge to a simulation. It has been used by L. Tian et al [17, pp. 354-362] to solve a 4 GBM model on photovoltaic power generation with carbon market linkage in China, by L. Zhu & Y. Fan [18, pp. 4320-4333] for a carbon capture and storage investment or by L. Zhu [19, pp. 585-593] in Nuclear energy project.

Regarding the valuation of Real option on photovoltaic investment, the main contributions come from Eduardo Alejandro Martinez-Cesena et al. [20, pp. 2087-2094] who analyze a defer option in 2010 in UK with a decreasing stochastic investment cost due to the technological progress. The option was paying off in 2015 to wait the availability of a new technology (organic thin film) that allow to reduce the cost and improving the energy production. Again, on the investment cost side, C. Jeon & J. Shin [21, pp. 447-457] used a stochastic learning curve with 2 factors to study the influence at long term of technological changes in solar panel technology. The conclusion is that the reduction of investment will continue in the coming years (to 2030) and that the subsidy of the governments plays a key role in this evolution. Another contribution comes from M.M. Zhang et al. [22, pp. 213-226] where an LSM model was used to value a solar panel project in China with variables on the electricity price, investment cost, benefits generated by the sold of CO^2 reduction quota and corporate tax policy. The results tend to show that the low electricity price at this time (2010) and the insufficiency of subsidy lead to a non-optimal investment, those variables should play a significative impact in the value of the option. Those researches insist on the influence of main variables which are investment cost, electricity price and subsidy policy. It's why the Belgian context of solar panel is so interesting because the uncertainty of the subsidy legislation varies a lot between the regions and through the time, in a context of growing electricity cost due to nuclear energy dependence and the reduction of investment cost that occurred with the technological progress.

a) Variables and initial hypothesis of the model

Multiple kinds of model will now be considered:

1° Analytical model with 1 GBM

2° Analytical model with 2 GBM, based on McDonald & Siegel solution [23]

3° Least-Square Monte Carlo model applied to the Belgian context

The following variables will be considered depending on the complexity of the model

General information and location of the installation:

The investment project considered will be located at Mouscron for the Walloon region which is a city at the corner of Wallonia, Flanders and France. For the Flemish region, it's located at Spiere, a city next to Mouscron (same electricity production and solar irradiance). It's a straightforward way to compare the 2 different situations. For the Brussels region, the same irradiance will be assumed to compare the results.[43]

The model of solar panel will be a monocrystalline panel (technical characteristic will be described later). The life of the panel is 20 years (the average life of a solar panel is comprised between 20 and 25 years, 20 years is the life of the guarantee) and the option maturity of 10 years to be in line with the vision of a householder. The installation will have a power of 3 kWc, a size adapted to the consumption of the common householder in Belgium (3,5 kWh) in order to have the largest auto-consumption factor. No taxes are applied on the selling of CV or electricity.[44] The number of CV granted will based on the maximal value presented on the different tables above.

The kind of real option analyzed will exclusively be a right to wait, other kinds are not relevant as an incremental option because a solar panel can be installed in a few days.

Electricity price variable (E):

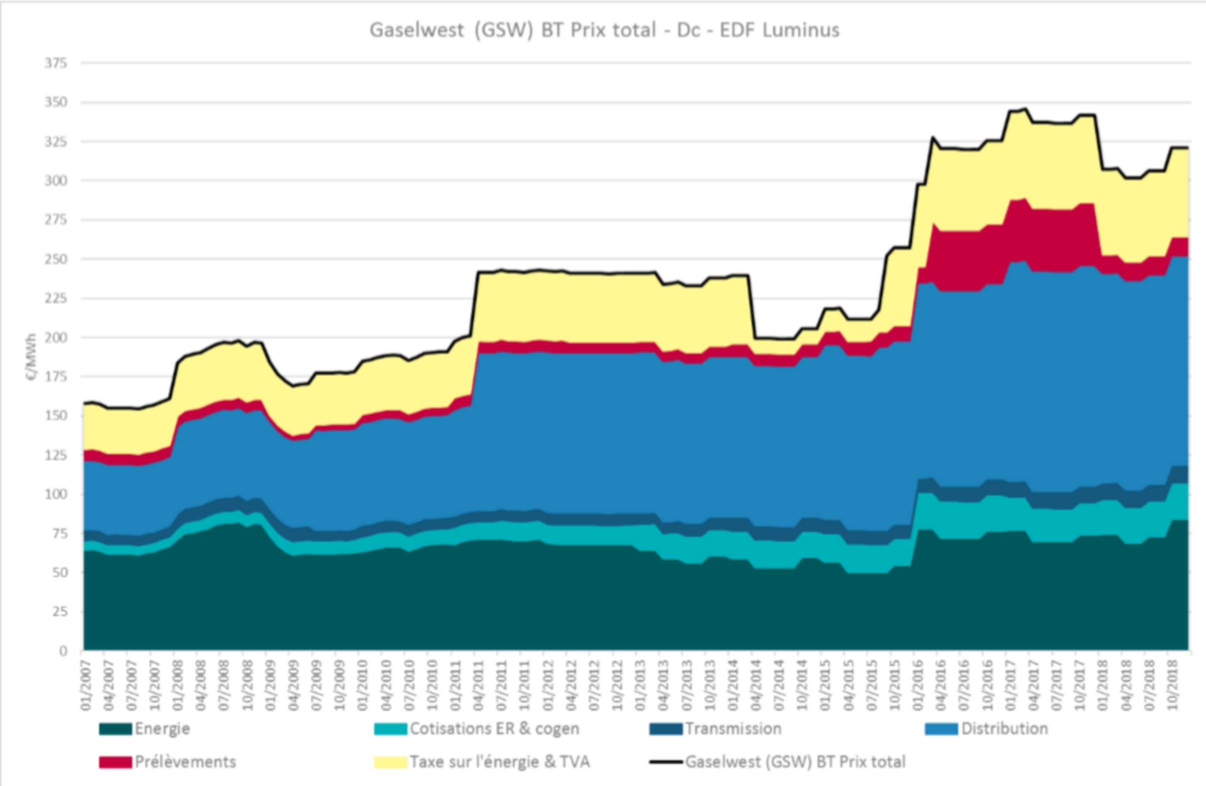
Based on the rapport of CREG (Federal regulator of energy) [24], the common householder has a consumption of 3.500 kWh per year (DC 2v, CREG methodology) [24, p. 12]. For different reasons, electricity price is different between the region. [24, p. 126]. Tax and energy policies depend of the regions and the price itself is affected by multiple components. The next figure represents the evolution of the components impact. In 2011 and 2016, distribution costs increase a lot. VAT (in yellow) decreased from 21% to 6% in 04/2014 to be increased at the previous level (21%) in 08/2015. On the Belgium level, price has risen about 61,59% between 2007 and 2018, with an increase of 3,33% for the year 2018.

[43] The difference with the real life is assumed to be small.
[44] https://www.cwape.be/?dir=3.4.01, consulted on 27/04/2019

Figure 16 – Price evolution with the components impact for one of the main suppliers (EDF Luminus)

The price of electricity increased about 62% in 12 years in Belgium, mainly due to the growing distribution costs

Graphique 9 : Evolution des 6 principales composantes du prix total, Dc 2v, Gaselwest – EDF Luminus

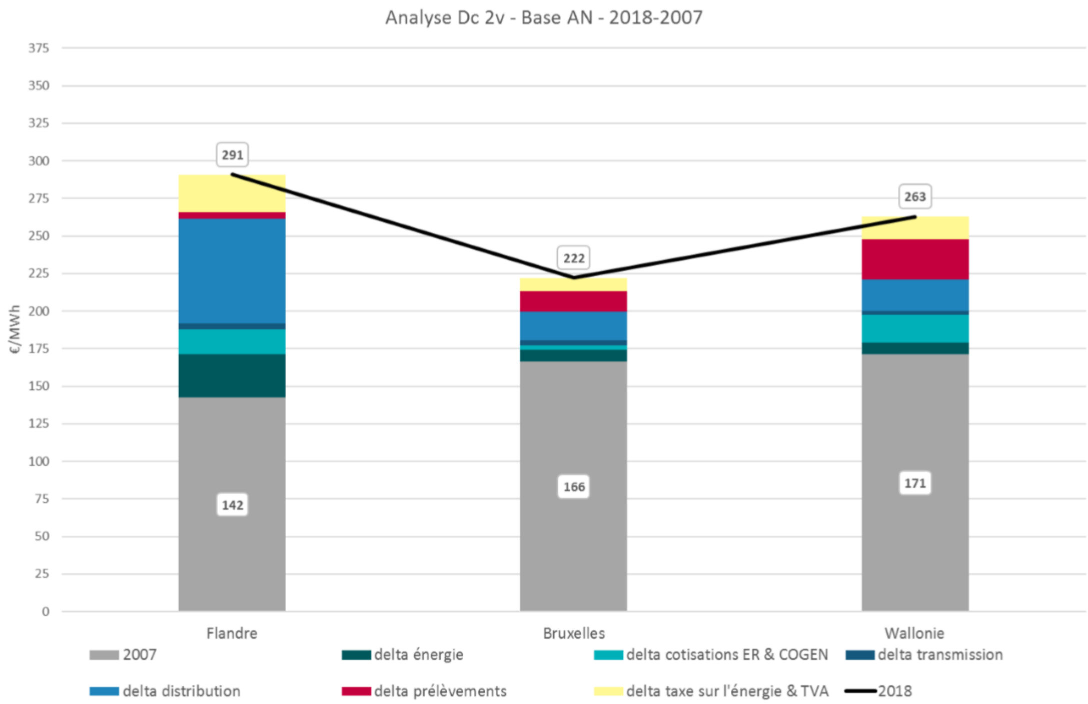


The following table provides information's about the region situation at the end of 2018.

Region	2007 (January)	2018 (December)	Variation	%	% annual (12 years)[45]
Flanders	142 €/MWh	291 €/MWh	148,31 €/MWh	104,93%	6,16%
Brussels	166 €/MWh	222 €/MWh	55,83 €/MWh	33,73%	2,45%
Wallonia	171 €/MWh	263 €/MWh	91,57 €/MWh	53,80%	3,65%

[45] Average increase calculated as follow: $\sqrt[(2018-2007)+1]{\frac{Value_{2018}}{Value_{2007}}} - 1$

Figure 17 – Evolution of electricity price (by MWh) for the 3 regions



Based on those data's, some hypothesis will be made about the variable's values:

Initial price:

> 220, 260 and 300 (€/ MWh) which represent the starting situation of the 3 regions.

Trend of the electricity price:

> 2%, 3%, 4% and 7% which represent the historical trend, the price is not assumed to decrease in the coming years due to the CV debt in Wallonia or the climate transition policy.

Volatility of the electricity price:

> Using the data on the figure of EDF Luminus price, the standard error of the price changes gives a value of 0,1865 which will be round to 0,2 for simplicity.

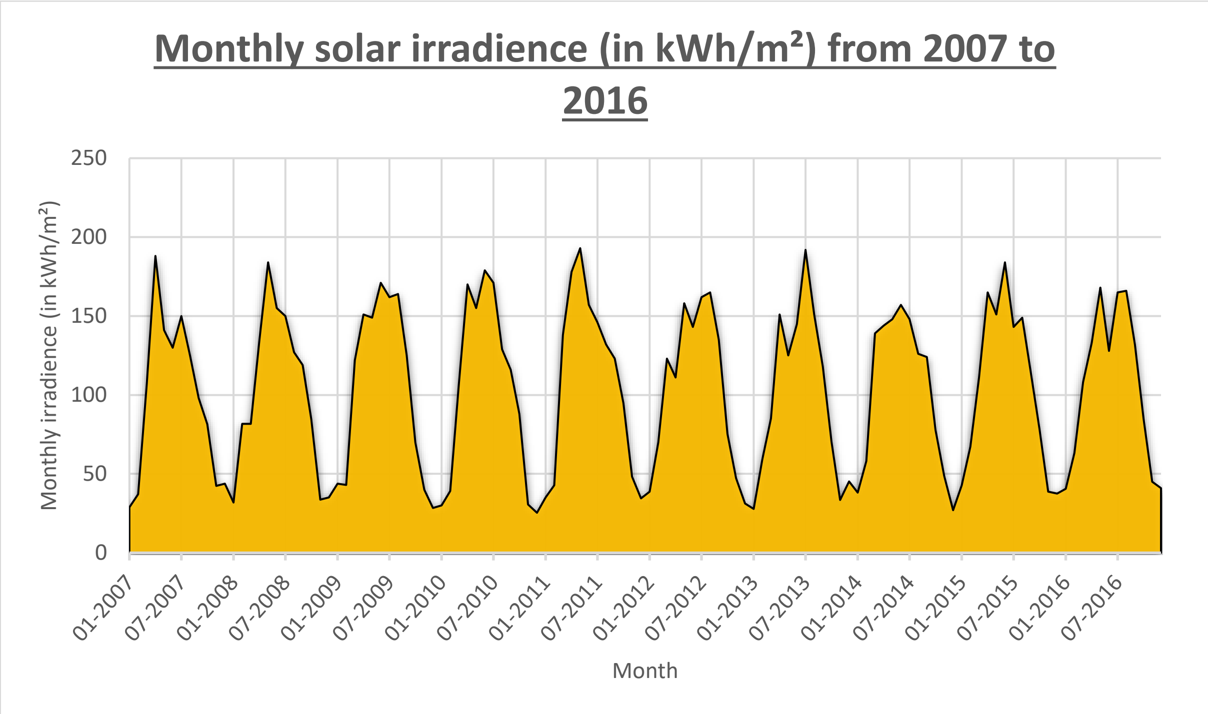
No mean-reverting process is assumed on the total price, the Belgium context influences more the final price than the commodity variation. Those hypotheses are lower than the forecasting of the CWAPE [10, p. 93], comparisons will be made if necessary.

Electricity produced variable (Q):

Based on data from the European Union research database[46], a mean-reverting process will be estimated because the data are very seasonal. This database gives the monthly irradiation (in kWh/m²) which represents the optimal condition of a solar panel: south-east to -south-west orientation, gradient at 36°. The model will consider a solar panel installation in the optimal conditions of production.[47]

Figure 18 - Monthly solar irradiance

The monthly irradiance in Belgium varies from 30 kWh/m² in the winter to 190 in the summer.
The mean on a year is about 107



1 solar panel[48] has a size of 0,942 x 1,640, it corresponds to 1,54 m² per panel. The conversion factor of solar energy into electricity of the panel is 20%. Each panel has a kWc of 1,54*20% = 0,308. The total capacity of the installation must be 3 kWc, so 9,74 (3/0,308) solar panels are necessary, this amount is rounded to 10. The system loss is assumed to be 14%[49]. The total installation produces 0,860 kWh for each kWc in optimal conditions. The values of the database will be so multiplied by 0,86*3 (kWh*kWc). A negative trend (Loss_factor) of -0,5%/year will be applied to represent the loss of production due to the age of the installation[50].

[46] http://re.jrc.ec.europa.eu/pvgis/

[47] (Located in Mouscron; latitude: 50,724°; Longitude: 3,314°; Radiation database: PVGIS-CMSAF, data extracted on 17/05/2019)

[48] Technical characteristic available at https://forevergreen-products.co.uk/product/290w-all-black-monocrystalline-b3/

[49] As recommended on the database website

[50] Same value used that the CWAPE methodology for Qualiwatt

$$Q_t = \left(100\% - Loss_{factor}\right) * (Q_{t-1} + dQ) \quad (24)$$

The mean-reverting process is set as:

$$dQ = \bar{\eta} * (\bar{Q} - Q) * dt + \sigma * dW_t \quad (25)$$

The coefficients will be valued through an analytical solution of Maximum likelihood estimation of the process[51]:

$$S_X = \sum_{i=1}^{n} S_{i-1}$$
$$S_Y = \sum_{i=1}^{n} S_i$$
$$S_{XX} = \sum_{i=1}^{n} S_{i-1}^2 \quad (26)$$
$$S_{XY} = \sum_{i=1}^{n} S_{i-1} * S_i$$
$$S_{YY} = \sum_{i=1}^{n} S_i^2$$

Where S represent the monthly average irradiance

Parameters of the mean-reverting process are given by:

$$\bar{Q} = \frac{\left(S_Y - e^{-\lambda*\delta} * S_X\right)}{n * (1 - e^{-\lambda*\delta})}$$

$$\bar{\eta} = -\frac{1}{\delta} * \ln \frac{\left(S_{XY} - \bar{Q}*S_X - \bar{Q}*S_Y + n*\bar{Q}^2\right)}{\left(S_{XX} - 2*\bar{Q}*S_X + n*\bar{Q}^2\right)}$$

$$\alpha = e^{-\lambda*\delta}$$

$$\hat{\sigma}^2 = \frac{1}{n} * (S_{YY} - 2 * \alpha * S_{XY} + \alpha^2 * S_{XX} - 2 * \bar{Q} * (1 - \alpha) * (S_Y - \alpha * S_X) + n * \bar{Q}^2 * (1 - \alpha)^2) \quad (27)$$

$$\sigma^2 = \hat{\sigma}^2 * \frac{2 * \lambda}{1 - \alpha^2}$$

Where $\delta = 1/12$ as the data are in the monthly format

[51] A guide to the coefficient estimation can be found at: https://www.statisticshowto.datasciencecentral.com/wp-content/uploads/2016/01/Calibrating-the-Ornstein.pdf , consulted on 19/05/2019

Coefficients are equal to:

- $\bar{Q} = 104{,}96$
- $\bar{\eta} = 3{,}1033$
- $\sigma = 126{,}2737$

The initial value of Q will be fixed at the mean value (104,96) to avoid timing strategy.

Investment cost (I):

Using the data of the CWAPE to compute Qualiwatt subsidy, the coefficient "I" has a trend about -11,5%/year and a volatility of 0.075, it will also be used for the model. The price of a solar panel is supposed to decrease in the time based on the future technology innovation.[52] The initial investment cost will be fixed at 6.500€ based on an IKEA simulation, provided here as an example[53].

Votre estimation

Ce tableau indique approximativement combien vous gagnez et économisez. Pour un calcul précis, adapté à votre habitation, demandez un devis. Sur quels éléments se base ce calcul?

Option	BASIC	PLUS	DESIGN
Nombre de panneaux	14	13	14
Prix public, sans la réduction de 15% pour les membres IKEA FAMILY	€5894	€6418	€7578
Prix	€5050	€5500	€6500
Estimation de la baisse de votre facture d'électricité	60%	68%	60%
Panneaux	Blue 270W	Black 300W	Sunstation 270W
Onduleurs	GoodWe	GoodWe	GoodWe

The cost of the inverter (only one for the installation) is assumed to be stable at 250€ (CWAPE data). It happens every 10 years, one time by installation for a life of 20 years. By simplicity, it will be actualized in the investment cost based on the following formula:

$$\frac{250}{(1+r)^{10}} \quad (28)$$

Maintenance cost:

It will be approximated to be 0,75% of the initial investment cost as in the CWAPE methodology.

[52] Some new technologies can be found on this business press article: https://www.bfmtv.com/economie/cette-innovation-rend-les-panneaux-solaires-plus-legers-et-bon-marche-1632290.html
[53] Other cost simulations have been conducted and give the same value.

Walloon Prosumer Tax (PRO_WL):

The value will be fixed at 78€/kWe (ORES Mouscron price for the planned tax of 2020). Auto-consumption is assumed to be equal to 37,76% of the electricity production without smart meter, for simplicity the value will be based on the total kWc of the installation.

Flemish Prosumer Tax (PRO_FL):

The value will be fixed at 110€ (2019 value for Gaselwest). The value will be based on the total kWc of the installation.

Brussels Certificat Vert (CV)

Using historical price of CV at Brussels, the initial price will be fixed at 96€/CV (value for the end of 2018), with a small trend of 0,3%/year and a volatility of 0,03. Price will be constrained to the range [65€ ; 100€] as explained previously. 1CV is granted by 1.000 kWh.

Interest rate (r)

As the probable investor is expected to be a household owner, CAPM theory cannot be applied to determine the capital cost. Instead, the interest rate of the savings account (close to 0%) or the interest rate on a green loan (2-3%) will be applied.
Multiple value will be tested: 0,5%; 1%; 3% and 5%

For the risk neutral process (model 1 and 2):

$$\delta = \mu - \alpha = r + \frac{(r_M - r)}{\sigma_M} * \sigma * \rho_{E;M} - \alpha$$

- α is the trend of the process
- σ is the volatility of the process
- r_M is the expected return on the market
  Eurostoxx 50 is used as the market of reference because the index is more diversified than the BEL20 index.
  Using function "Return.calculate"[54] on the monthly data of yahoo finance, the median return is about 0,3928%/month on the period 01-01-2000/31-12-2018
- σ_M is also valued on the same basis and as a value of 0,018/year
- The correlation term is fixed at:
  - +0.175 between CV and M, based on quarterly data (2011-Q2/2018-Q4)
  - -0.04 between E and M (approximated on yearly data of Flemish price of electricity and yearly data of Eurostoxx50).
  - +0.32 between I and M, based on quarterly data (2014-Q2/2016-Q2)

[54] https://www.rdocumentation.org/packages/PerformanceAnalytics/versions/1.5.2/topics/Return.calculate , consulted on 17/05/20119

To keep the model simple as possible, interest rate and volatility will be kept stable even it's not the case in real life. The difference is not supposed to influence the results as multiple values will be tested.

Some ranges are used to avoid that the stochastic process provides abnormal values. For electricity price, the range is [100€ ; 700€] which is equivalent to a price lower than the situation in 2007 and an increase between 200 and 300% from the initial situation. For the solar irradiance, the range is [20 ; 200] which correspond to the historical minimum and maximum observed. For the investment cost, the range is [500€ ; 7000€] which corresponds to a minimal price per panel of 50€ and an expensive installation on the 01/01/2019.

b) Summary of the variable:

General information:		
Life of the installation (guarantee)	20 years	
Maturity of the option	10 years	
Power of the installation	3 kWc	
Electricity price variable (E):		range [100€ ; 700€]
Initial value:	220, 260 and 300	
Trend:	2%, 3%, 4% and 7%/year	
Volatility:	0,2/year	
Electricity produced variable (Q):		range [20 ; 200]
Initial value	104,96	
$\bar{Q}$	104,96	
$\bar{\eta}$	3,1033	
$\hat{\sigma}$	126,2737	
Loss factor	0,5%/year	
Investment cost (I):		range [500€ ; 7000€]
Initial value:	6.500,00€	
Trend:	-11,5%/year	
Volatility:	0,075/year	
Inverter cost:	$\frac{250}{(1+r)^{10}}$	Summed with initial I
Maintenance cost:	0,75% * I	
Walloon Prosumer Tax (PRO_WL):		
Initial value:	78€ /kWe	Valued as (1-37,76%) of the kWc
Trend:	1%/year	
Auto consumption factor:	37,76%	
Flemish Prosumer Tax (PRO_FL):		
Initial value:	110€ /kW of kVA	Valued as initial kWc
Trend:	1%/year	
Brussels Certificat Vert (CV):		
Initial value:	96€/CV	
Trend:	0,3%/year	
Volatility:	0,03/year	range

		[65€ ; 100€]
CV number	3 CV/ 1.000 kWh	
Interest rate (r)	0,5%; 1%; 3% and 5%/year	

3. Model 1: Perpetual and basic option with 1 GBM

The first model is based on the methodology developed at the point 4 of the theorical part. One geometric Brownian motion (GBM) will be assumed on the electricity price in an exotic way because it will be multiplied by a constant and average production level. The project will yield a perpetual profit of G when the investment is achieved. No maintenance cost as correlation between the variables is assumed to keep the model simple.

Model 1: Solution with a simple model of 1 GBM

Definition of the variables

$\overline{PR}$: assume an exogenous and average production of energy which is constant

E: the price of the electricity following a Brownian motion

Process equation of E (Geometric Brownian Motion):

$$d\,E = \alpha_E * E * dt + \sigma_E * E * dW_t \qquad (29)$$

G: gains generated from the production of energy

$$\mathrm{G} = \mathrm{E} * \overline{PR} \qquad (30)$$

I: the fixed cost of the investment, which represents the sunk cost as the solar panels lose all their value while they are installed on a roof.

F: the global value of the project

No correlation is assumed between E and I

The goal is to find an optimal level G^* that must reach the cash-flows (G) to provide the highest profitability based on the actual information and present situation.

Gains from the production of energy can be set equal to:

$$d\,G = d\,E * \overline{PR} \qquad (31)$$

Or

$$d\,G = \alpha_E * E * \overline{PR} * dt + \sigma_E * \overline{PR} * E * dW_t \qquad (32)$$

Another to define G is by using the integral form [8, p. 150], it represents the ongoing present value at time 0 of an infinite flow which consists in E*$\overline{PR}$:

$$G_0 = \int_0^{\infty} \delta * G_0 * e^{(\mu-\delta)*t} * e^{-r*t} dt \tag{33}$$

Variance of d G:

$$(d\,G)^2 = \alpha_E^2 * E^2 * \overline{PR}^2 * dt^2 + \sigma_E^2 * \overline{PR}^2 * E^2 * dW_t^2 + 2 * \alpha_E * E * \overline{PR} * \sigma_E * \overline{PR} * E * dW_t * dt \tag{34}$$

Simplifying,

$$(d\,G)^2 = \sigma_E^2 * \overline{PR}^2 * E^2 * dW_t^2 + 2 * \alpha_E * E * \overline{PR} * \sigma_E * \overline{PR} * E * dW_t * dt \tag{35}$$

It can be shown that the stochastic increment (dW_t) is equal to the root of dt

$$dW_t = \sqrt{dt} \tag{36}$$

$$(d\,G)^2 = \sigma_E^2 * \overline{PR}^2 * E^2 * dt + 2 * \alpha_E * E * \overline{PR} * \sigma_E * \overline{PR} * E * dW_t * dt \tag{37}$$

By relation with equation 36:

$$(d\,G)^2 = \sigma_E^2 * \overline{PR}^2 * E^2 * dt + 2 * \alpha_E * E * \overline{PR} * \sigma_E * \overline{PR} * E * dt^{\frac{3}{2}} \tag{38}$$

Again the term $dt^{\frac{3}{2}}$ turn faster to 0 than other members and can be cancelled:

$$(d\,G)^2 = \sigma_E^2 * \overline{PR}^2 * E^2 * dt \tag{39}$$

Developing the components of differential equation of F following Ito's lemma [8, pp. 79-82]

$$d\,F(G,t) = \frac{\partial F}{\partial t} dt + \frac{\partial F}{\partial G} dG + \frac{1}{2} * \frac{\partial^2 F}{\partial^2 G} dG^2 + \frac{\partial^2 F}{\partial^2 t} dt^2 + \frac{\partial^2 F}{\partial G \partial t} dG\,dt + \cdots \tag{40}$$

The last 2 terms can be neglected because dt^2 and $dG\,dt$ turn faster to 0 than others.

$$d\,F(G,t) = \frac{\partial F}{\partial t} dt + \frac{\partial F}{\partial G} dG + \frac{1}{2} * \frac{\partial^2 F}{\partial^2 G} dG^2 \tag{41}$$

Substituting dG^2 by its variance,

$$d\,F(G,t) = \frac{\partial F}{\partial t} dt + \frac{\partial F}{\partial G} dG + \frac{1}{2} * \frac{\partial^2 F}{\partial^2 G} * var(dG) \tag{42}$$

dG is given by equation 32 and the variance by equation 39

$$d\,F(G,t) = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial G} * (\alpha_E * E * \overline{PR} * dt + \sigma_E * \overline{PR} * E * dW_t) + \frac{1}{2} * \frac{\partial^2 F}{\partial^2 G} * var(\alpha_E * E * \overline{PR} * dt + \sigma_E * \overline{PR} * E * dW_t) \quad (43)$$

$$d\,F(G,t) = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial G} * (\alpha_E * E * \overline{PR} * dt + \sigma_E * \overline{PR} * E * dW_t) + \frac{1}{2} * \frac{\partial^2 F}{\partial^2 G} * \sigma_E^2 * \overline{PR}^2 * E^2 * dt \quad (44)$$

Regrouping terms implies,

$$d\,F(G,t) = \left[\frac{\partial F}{\partial t} + \alpha_E * E * \overline{PR} * \frac{\partial F}{\partial G} + \frac{1}{2} * \frac{\partial^2 F}{\partial^2 G} * \sigma_E^2 * \overline{PR}^2 * E^2\right] * dt + \frac{\partial F}{\partial G} * \sigma_E * \overline{PR} * E * dW_t \quad (45)$$

The first member into brackets represent the determinist part and the second, the stochastic ones.

Risk-free portfolio and arbitrage conditions [7, p. 41]

To construct a risk-free portfolio, we need a position in the project (F) and a short position in the underlying asset (G), which consists in a hedge of the investment. Δ_1 is the size of the position to obtain a perfect hedge with G on the underlying F.

$$Port = F - \Delta_1 G \quad (46)$$

And in the increment format,

$$d\,Port = d\,F - \Delta_1\,d\,G \quad (47)$$

Developing dF:

$$d\,Port = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial G}dG + \frac{1}{2} * \frac{\partial^2 F}{\partial^2 G}dG^2 - \Delta_1\,d\,G \quad (48)$$

Replacing dG^2 by its variance,

$$d\,Port = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial G}dG + \frac{1}{2} * \frac{\partial^2 F}{\partial^2 G} * \sigma_E^2 * \overline{PR}^2 * E^2 * dt - \Delta_1\,d\,G \quad (49)$$

Regrouping terms with dG in common:

$$d\,Port = \frac{\partial F}{\partial t}dt + \frac{1}{2} * \frac{\partial^2 F}{\partial^2 G} * \sigma_E^2 * \overline{PR}^2 * E^2 * dt + \left(\frac{\partial F}{\partial G} - \Delta_1\right) * d\,G \quad (50)$$

d G represents the risk driver, it must set equal to 0 to be risk free:

$$\left(\frac{\partial F}{\partial G} - \Delta_1\right) * \; d\, G = 0 \tag{51}$$

To obtain equality in the equation, the value of Δ_1 must be equal to:

$$\Delta_1 = \frac{\partial F}{\partial G} \tag{52}$$

$$d\ Port = \frac{\partial F}{\partial t} dt + \frac{1}{2} * \frac{\partial^2 F}{\partial^2 G} * \sigma_E^2 * \overline{PR}^2 \; * E^2 * \; dt + \left(\frac{\partial F}{\partial G} - \Delta_1\right) * \; d\, G \tag{53}$$

To obtain a solution, the insertion of the term δ_E is need. It's the equivalent to a dividend payoff[55] to maintain the option alive (for more details see 4.b) of the theorical part). It can be computed as follow [8, p. 155]

$$\delta_E = \mu_E - \alpha_E = r + \phi * \sigma * \rho_{E;M} - \alpha_E \tag{54}$$

$$d\ Port = \frac{\partial F}{\partial t} dt + \frac{1}{2} * \frac{\partial^2 F}{\partial^2 G} * \sigma_E^2 * \overline{PR}^2 \; * E^2 * \; dt + \left(\frac{\partial F}{\partial G} - \Delta_1\right) * \; d\, G + \delta_E * \Delta_1 * E * \overline{PR} * dt \tag{55}$$

Substituting the value of Δ_1 into the equation 55:

$$d\ Port = \frac{\partial F}{\partial t} dt + \frac{1}{2} * \sigma_E^2 * \overline{PR}^2 \; * E^2 * \frac{\partial^2 F}{\partial^2 G} * \; dt + \delta_E * E * \overline{PR} * \frac{\partial F}{\partial G} * dt \tag{56}$$

Risk-free portfolio

This portfolio must give a risk-free return (r) at each time increment dt:

$$d\ Port = r * Port * dt \tag{57}$$

Using Port with the equation 46,

$$\frac{\partial F}{\partial t} dt + \frac{1}{2} * \sigma_E^2 * \overline{PR}^2 \; * E^2 * \frac{\partial^2 F}{\partial^2 G} * \; dt + \delta_E * E * \overline{PR} * \frac{\partial F}{\partial G} * dt = r * (F - \Delta_1 G) * dt \tag{58}$$

Bellman's equation (optimality condition):

$$\frac{\partial F}{\partial t} dt + \frac{1}{2} * \sigma_E^2 * \overline{PR}^2 \; * E^2 * \frac{\partial^2 F}{\partial^2 G} * \; dt + \delta_E * E * \overline{PR} * \frac{\partial F}{\partial G} * dt - r * (F - \Delta_1 G) * dt = 0 \tag{59}$$

[55] Even if the underlying doesn't provide any dividends.

Simplifying the dt:

$$\frac{\partial F}{\partial t} + \frac{1}{2} * \sigma_E^2 * \overline{PR}^2 * E^2 * \frac{\partial^2 F}{\partial^2 G} + \delta_E * E * \overline{PR} * \frac{\partial F}{\partial G} - r * (F - \Delta_1 G) = 0 \tag{60}$$

Substituting Δ_1 and G by their values (equation 52 and 30):

$$\frac{\partial F}{\partial t} + \frac{1}{2} * \sigma_E^2 * \overline{PR}^2 * E^2 * \frac{\partial^2 F}{\partial^2 G} + \delta_E * E * \overline{PR} * \frac{\partial F}{\partial G} - r * \left(F - \frac{\partial F}{\partial G} * E * \overline{PR}\right) = 0 \tag{61}$$

Regrouping terms,

$$\frac{\partial F}{\partial t} + \left(\frac{1}{2} * \sigma_E^2 * \overline{PR}^2 * E^2 * \frac{\partial^2 F}{\partial^2 G}\right) + \left((r - \delta_E) * E * \overline{PR} * \frac{\partial F}{\partial G}\right) - r * F = 0 \tag{62}$$

Converting $\frac{\partial^2 F}{\partial^2 G}$ into F'' and $\frac{\partial F}{\partial G}$ into F'

$$\frac{\partial F}{\partial t} + \left(\frac{1}{2} * \sigma_E^2 * \overline{PR}^2 * E^2\right) * F'' + \left((r - \delta_E) * E * \overline{PR}\right) * F' - r * F = 0 \tag{63}$$

The term with "t" is neglected to give a perpetual option:

$$\left(\frac{1}{2} * \sigma_E^2 * \overline{PR}^2 * E^2\right) * F'' + \left((r - \delta_E) * E * \overline{PR}\right) * F' - r * F = 0 \tag{64}$$

As $E * \overline{PR}$ is equal to G,

$$\left(\frac{1}{2} * \sigma_E^2 * G^2\right) * F'' + \left((r - \delta_E) * G\right) * F' - r * F = 0 \tag{65}$$

Solution to the Bellman's equation:

We try a general solution for a differential equation with geometric Brownian motion as explained in the theorical part:

$$F = A * G^\beta \tag{66}$$

Applying this function, it gives:

$$\left(\frac{1}{2} * \sigma_E^2 * G^2\right) * \left(A * G^\beta\right)'' + \left((r - \delta_E) * G\right) * \left(A * G^\beta\right)' - r * \left(A * G^\beta\right) = 0 \tag{67}$$

Taking the derivatives of the solution,

$$\left(\frac{1}{2} * \sigma_E^2 * G^2\right) * \left[\beta * (\beta - 1) * \left(A * G^{\beta-2}\right)\right] + \left((r - \delta_E) * G\right) * \left[\beta * \left(A * G^{\beta-1}\right)\right] - r * \left(A * G^{\beta}\right) = 0 \tag{68}$$

Simplifying with power of G:

$$\left(\frac{1}{2} * \sigma_E^2\right) * \left[\beta * (\beta - 1) * \left(A * G^{\beta}\right)\right] + (r - \delta_E) * \left[\beta * \left(A * G^{\beta}\right)\right] - r * \left(A * G^{\beta}\right) = 0 \tag{69}$$

Dividing by $A * G^{\beta}$:

$$\left(\frac{1}{2} * \sigma_E^2 * \beta * (\beta - 1)\right) + \left((r - \delta_E) * \beta\right) - r = 0 \tag{70}$$

Regrouping terms,

$$\left(\frac{1}{2} * \sigma_E^2 * (\beta^2 - \beta)\right) + \left((r - \delta_E) * \beta\right) - r = 0 \tag{71}$$

Dividing by σ_E^2 and again regrouping terms:

$$0.5 * \beta^2 + \left(\frac{(r - \delta_E)}{\sigma_E^2} - 0.5\right) * \beta - \frac{r}{\sigma_E^2} = 0 \tag{72}$$

Roots and solution of the equation:

To find the roots of β, we apply the formula:

$$\beta = \frac{-b \pm \sqrt{b^2 - 4 * a * c}}{2 * a}$$

$$a = 0.5$$

$$\mathrm{b} = \left(\frac{(r-\delta_E)}{\sigma_E^2} - 0.5\right) \tag{73}$$

$$c = -\frac{r}{\sigma_E^2}$$

We obtain 2 roots:

$$\beta_1 = \frac{-\left(\frac{(r - \delta_E)}{\sigma_E^2} - 0.5\right) + \sqrt{\left(\left(\frac{(r - \delta_E)}{\sigma_E^2} - 0.5\right)\right)^2 - 4 * 0.5 * -\frac{r}{\sigma_E^2}}}{2 * 0.5} \tag{74}$$

$$\beta_1 = -\left(\frac{(r-\delta_E)}{\sigma_E^2} - 0.5\right) + \sqrt{\left(\left(\frac{(r-\delta_E)}{\sigma_E^2} - 0.5\right)\right)^2 - 4 * 0.5 * - \frac{r}{\sigma_E^2}} \quad (75)$$

And

$$\beta_2 = -\left(\frac{(r-\delta_E)}{\sigma_E^2} - 0.5\right) - \sqrt{\left(\left(\frac{(r-\delta_E)}{\sigma_E^2} - 0.5\right)\right)^2 - 4 * 0.5 * - \frac{r}{\sigma_E^2}} \quad (76)$$

The resultant solution of this process gives:

$$F = A_1 * G^{\beta_1} + A_2 * G^{\beta_2} \quad (77)$$

First option: Call on the benefit part of the project

The goal is to invest, it means taken a long position in the project, this kind of option consists in a call. For this option, only the first member is positive due to the boundary conditions (summary of those boundaries is provided on the next table), the second member is on its side equal to 0.

Boundary condition	Call
1°	F (0) = 0
2°	F (G*) = G* - I
3°	F'(G*) = 1
Results	$A_2 = \beta_2 = 0$ Only keep the positive root

Solution for the first member:

$$A_1(G^*)^{\beta_1} = G^* - I \quad (78)$$

$$A_1 = \frac{G^* - I}{(G^*)^{\beta_1}}$$

$$\beta_1 = 0.5 - \left(\frac{(r-\delta_E)}{\sigma_E{}^2}\right) + \sqrt{\left(\frac{r}{\sigma_E{}^2} - 0.5\right)^2 + \frac{2 * r}{\sigma_E{}^2}} \quad (79)$$

$$A_2 = 0$$

Optimal level of G where X is the strike of the option:

$$G^* = \frac{\beta_1}{\beta_1 - 1} * I \tag{80}$$

When G reaches this value (trigger level), it will be optimal to invest in the project by paying an investment cost of I.

One specificity must be added now because the project is a series of infinite flow, the value of G at time t [8, p. 144] can be rewritten as:

$$G_t = \varepsilon \int_t^{\infty} E_s * \overline{PR} * e^{-r*(s-t)} ds = \frac{(E_t * \overline{PR})}{r - \alpha_E} \tag{81}$$

This formula is different of G at time 0 as the option value is dynamic. The solution just become [8, p. 145]:

$$G^* = (E * \overline{PR})^* = \left(r + \frac{1}{2} * \sigma_E^2 * \beta_1\right) * I \tag{82}$$

It means that when the future profits are uncertain, the threshold $(E * \overline{PR})^*$ must exceed the user cost of capital. The relation comes from the deterministic part of capital cost (r*I) with an additional term of $\frac{1}{2} * \sigma_E^2 * \beta_1$ which represents the additional cost of the risk. This equation will not be used in the results as it implies an infinite series of cash-flows which is not the case of a solar panel (20 years).

4. Model 2: Model with correlation and 2 GBM

This model is based on the work of McDonald & Siegel [23], again in an exotic way. 2 GBM processes are considered, the electricity price (E) as in the model 1 and a stochastic investment cost (I). No maintenance cost is used to keep the model simple.

Model 2: Solution by a similar model based on the work of McDonald & Siegel [23]

Definition of the variables

$\overline{PR}$: assume an exogenous production of energy that can be constant or determined by a function

E: price of the electricity following a Brownian motion

G: gains generated from the production of energy

G = E * $\overline{PR}$

I: stochastic cost of the investment, which is equal to the sunk cost, because the solar panel lose all its value while it's installed on a roof (see L below)

F: the value of the project

The process equations are:

$$d\,E = \alpha_E * E * dt + \sigma_E * E * dW_1 \tag{83}$$

$$d\,G = \alpha_E * E * \overline{PR} * dt + \sigma_E * \overline{PR} * E * dW_1 \tag{84}$$

In this model, I is given by the following Geometric Brownian motion:

$$d\,I = \alpha_I * I * dt + (\sigma_I * I * dW_2) \tag{85}$$

Variance:

Variance of a project (σ^2) with correlated variables:

$$Var(X - Y) = Var(X) + Var(Y) - 2 * Cov(X;Y) \tag{86}$$

By relation with equation 39, it gives in this case:

$$(d\,G)^2 = \sigma_E^2 * \overline{PR}^2 * E^2 * dt \tag{87}$$

$$(d\,I)^2 = \sigma_I^2 * I^2 * dt \tag{88}$$

Where the covariance between dG and dI is given by:

$$\begin{aligned} Cov(\,dG; dI) = dG * dI \\ &= \big(\alpha_E * E * \overline{PR} * dt \\ &+ (\,\sigma_E * \overline{PR} * E * dW_1)\big) \\ &* \big(\alpha_I * I * dt + (\sigma_I * I * dW_2)\big) \\ &= \big((\alpha_E * E * \overline{PR} * dt) * (\alpha_I * I * dt)\big) \\ &+ \big((\alpha_E * E * \overline{PR} * dt) * (\sigma_I * I * dW_2)\big) \\ &+ \big((\,\sigma_E * \overline{PR} * E * dW_1) * (\alpha_I * I * dt)\big) \\ &+ \big((\,\sigma_E * \overline{PR} * E * dW_1) * (\sigma_I * I * dW_2)\big) \end{aligned} \tag{89}$$

The first term is equal to 0 because $(dt)^2 = 0$ (from Ito's Lemma)

The second and third term are equal to 0 because (dt*dW) goes faster to 0 than other terms

$$dW = \sqrt{dt} * rnorm(0;1) \tag{90}$$

The results of two stochastic terms is given by:

$$\begin{aligned} dW_1 * dW_2 = \sqrt{dt} * rnorm(0;1) * \sqrt{dt} * rnorm(0;1) = dt * \rho_{W_1;W_2} \\ = dt * \rho_{G;I} \end{aligned} \tag{91}$$

It implies that the final covariance formula is,

$$Cov(\, dG; dI) = \big((\, \sigma_E * \overline{PR} \; * \; E * \; dW_1) * (\sigma_I * I * \; dW_2)\big) \\ = \; \sigma_E * \sigma_I * \overline{PR} * E * I * \rho_{G;I} * dt \tag{92}$$

When the boundary is independent of time (t), it's possible to obtain a solution where it's optimal to invest when V/F exceeds C* at the first time and wait before to invest other time. It's the same logic than the value of G* in the first model, represented here by C*.

Developing the components of differential equation of F following Ito's lemma [8, pp. 79-82]

Using Ito's lemma:

$$dF(G,I,t) = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial G}dG + \frac{\partial F}{\partial I}dI + \frac{1}{2} * \frac{\partial^2 F}{\partial^2 t}dt^2 + \frac{1}{2} * \frac{\partial^2 F}{\partial^2 G}dG^2 \\ + \frac{1}{2} * \frac{\partial^2 F}{\partial^2 I}dI^2 + \frac{1}{2} * \frac{\partial^2 F}{\partial G \partial I}dGdI + \frac{1}{2} * \frac{\partial^2 F}{\partial I \partial G}dIdG + \frac{1}{2} \\ * \frac{\partial^2 F}{\partial G \partial t}dGdt + \frac{1}{2} * \frac{\partial^2 F}{\partial I \partial t}dIdt + \cdots \tag{93}$$

The parts in yellow can be ignored because t goes faster to 0 than the other parameters.

$$dF(G,I,t) = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial G}dG + \frac{\partial F}{\partial I}dI + \frac{1}{2} * \frac{\partial^2 F}{\partial^2 G}dG^2 + \frac{1}{2} * \frac{\partial^2 F}{\partial^2 I}dI^2 \\ + \left[\frac{1}{2} * \frac{\partial^2 F}{\partial G \partial I}dGdI + \frac{1}{2} * \frac{\partial^2 F}{\partial I \partial G}dIdG\right] \tag{94}$$

The terms into bracket can be summed with Schwarz's lemma

$$dF(G,I,t) = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial G}dG + \frac{\partial F}{\partial I}dI + \frac{1}{2} * \frac{\partial^2 F}{\partial^2 G}dG^2 + \frac{1}{2} * \frac{\partial^2 F}{\partial^2 I}dI^2 \\ + \left[\frac{\partial^2 F}{\partial G \partial I}dGdI\right] \tag{95}$$

dG*dI is equal to the covariance between dG and dI

$$dF(G,I,t) = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial G}dG + \frac{\partial F}{\partial I}dI + \frac{1}{2} \\ * \left[\frac{\partial^2 F}{\partial^2 G}var(dG) + \frac{\partial^2 F}{\partial^2 I}var(dI) + 2\frac{\partial^2 F}{\partial G \partial I}cov(dGdI)\right] \tag{96}$$

Replacing variance and covariance by their values,

$$dF(G,I,t) = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial G}(\mu_E * E * \overline{PR} * dt + \sigma_E * \overline{PR} * E * dW_1) + \frac{\partial F}{\partial I}(\mu_I * I * dt + (\sigma_I * I * dW_2)) + \frac{1}{2} * \left[\frac{\partial^2 F}{\partial^2 G}(\sigma_E^2 * \overline{PR}^2 * E^2 * dt) + \frac{\partial^2 F}{\partial^2 I}(\sigma_I^2 * I^2 * dt) + 2\frac{\partial^2 F}{\partial G \partial I}(\sigma_E * \sigma_I * \overline{PR} * E * I * \rho_{G;I} * dt)\right] \quad (97)$$

$$dF(G,I,t) = \left[\frac{\partial F}{\partial t} + \frac{\partial F}{\partial G}(\mu_E * E * \overline{PR}) + \frac{\partial F}{\partial I}(\mu_I * I) + \frac{1}{2} * \left[\frac{\partial^2 F}{\partial^2 G}(\sigma_E^2 * \overline{PR}^2 * E^2) + \frac{\partial^2 F}{\partial^2 I}(\sigma_I^2 * I^2) + 2\frac{\partial^2 F}{\partial G \partial I}(\sigma_E * \sigma_I * \overline{PR} * E * I * \rho_{G;I})\right]\right] * dt + [(\sigma_E * \overline{PR} * E)dW_1 + (\sigma_I * I)dW_2] \quad (98)$$

The first member into brackets represent the determinist part and the second, the stochastic ones.

Risk-free portfolio and arbitrage conditions [7]

To obtain a perfect hedge of the portfolio (Port) it requires a position on the underlying F with a short position size of $\mathbf{\Delta_1}$ in G and $\mathbf{\Delta_2}$ in I. G and I represent the risk driver of the option and must neutralized to obtain a risk free portfolio (which avoid arbitrage opportunities).

$$Port = F - \Delta_1 G - \Delta_2 I \quad (99)$$

In the increment format,

$$dPort = d(F - \Delta_1 G - \Delta_2 I) \quad (100)$$

Substituting equation 97 into equation 99,

$$dPort = \left(\frac{\partial F}{\partial G} - \Delta_1\right)dG + \left(\frac{\partial F}{\partial I} - \Delta_2\right)dI + \left[\frac{1}{2} * \left[\frac{\partial^2 F}{\partial^2 G}(\sigma_E^2 * \overline{PR}^2 * E^2) + \frac{\partial^2 F}{\partial^2 I}(\sigma_I^2 * I^2) + 2\frac{\partial^2 F}{\partial G \partial I}(\sigma_E * \sigma_I * \overline{PR} * E * I * \rho_{G;I})\right] dt\right] \quad (101)$$

Where the solution of Δ_1 and Δ_2 are given by:

$$\Delta_1 = \frac{\partial F}{\partial G} \qquad \Delta_2 = \frac{\partial F}{\partial I} \quad (102)$$

The expected return of Electricity price (E) and Investment cost (I) can be defined as μ_E and μ_I. As it has been defined in the theorical part, it relies on the CAPM model where r is the free-risk rate, ϕ is the market price of risk[56], $\rho_{E;m}$ the correlation between E and the market (m) and $\rho_{I;m}$ the correlation between I and the market (m). It can be compared to the β as it represents the additional cost of risk compared to the market.

$$\mu_E = r + \phi * \rho_{E;m} * \sigma_E \tag{103}$$

$$\mu_I = r + \phi * \rho_{I;m} * \sigma_I \tag{104}$$

The measure of the dividend rate (δ_E and δ_I) is given by:

$$\delta_E = \mu_E - \alpha_E \tag{105}$$

$$\delta_I = \mu_I - \alpha_I \tag{106}$$

Where is α_E and $\alpha_I$is given by the trend of the Geometric Brownian Motion

To hold the short position, it is necessary to have:

$$(\Delta_1 * \delta_E * G + \Delta_2 * \delta_I * I)dt \tag{107}$$

As G and I give an additional return (δ_E and δ_I), it will lower the required rate of return to maintain the hedge of the option.

$$r * (F - (\Delta_1 * \delta_E * G + \Delta_2 * \delta_I * I)) * dt = \left(\frac{\partial F}{\partial G} - \Delta_1\right) dG + \left(\frac{\partial F}{\partial I} - \Delta_2\right) dI + \left[\frac{1}{2} * \left[\frac{\partial^2 F}{\partial^2 G}\left(\sigma_E^2 * \overline{PR}^2 * E^2\right) + \frac{\partial^2 F}{\partial^2 I}\left(\sigma_I^2 * I^2\right) + 2\frac{\partial^2 F}{\partial G \partial I}\left(\sigma_E * \sigma_I * \overline{PR} * E * I * \rho_{G;I}\right)\right] dt\right] \tag{108}$$

Bellman's equation:

At each increment of time dt, the equilibrium relation must hold, it means that the value of the portfolio

$$\boxed{\frac{1}{2} * \left[\frac{\partial^2 F}{\partial^2 G}\left(\sigma_E^2 * \overline{PR}^2 * E^2\right) + \frac{\partial^2 F}{\partial^2 I}\left(\sigma_I^2 * I^2\right) + 2\frac{\partial^2 F}{\partial G \partial I}\left(\sigma_E * \sigma_I * \overline{PR} * E * I * \rho_{G;I}\right)\right] + (r - \delta_E) * G * \frac{\partial F}{\partial G} + (r - \delta_I) * I * \frac{\partial F}{\partial I} - r * F = 0} \tag{109}$$

[56] $\phi = \frac{(r_M - r)}{\sigma_M}$; r_M is the expected return on the market; σ_M is the volatility of that return

The difficulty with this model is the presence of 2 GBM which transforms the conditions into a free-boundary problem. McDonald & Siegel have found a solution for 2 GBM, but an increase to 3 GBM could tend to an unsolvable analytical equation.

Solution to the Bellman's equation and boundary conditions [8, pp. 207-211]

Z is just the name of an option based on G; only used once to apply the boundary conditions. The detail about the boundary is just explained below.

$$F(G,I) = Z(G) - 1 = \frac{G}{\delta_E} - 1 \quad (110)$$

Figure 19 - Free Boundary problem [8, p. 208]

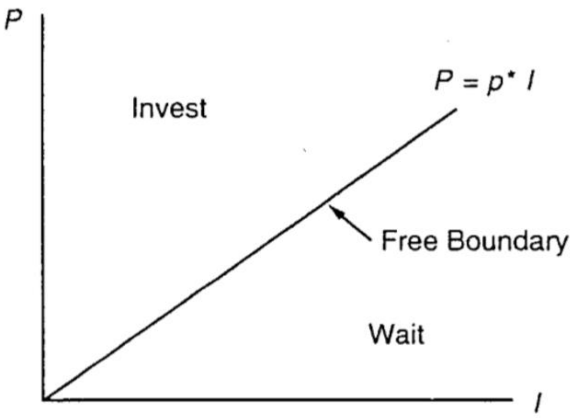


Figure 6.8. Investment with Price and Cost Uncertainty

The boundary between the two regions becomes a value-matching condition (between Wait and Invest area). As the 2 functions meet tangentially at the boundary, we have 2 smooth-pasting conditions.

$$\frac{\partial F}{\partial G}(G,I) = Z'(G) = \frac{1}{\delta_E} \quad (111)$$

$$\frac{\partial F}{\partial I}(G,I) = -1 \quad (112)$$

Optimal ratio to invest is formulated as C*

$$C = \frac{G}{I} = \frac{E * \overline{PR}}{I} \tag{113}$$

When the ratio $\frac{G}{I}$ reaches the optimal level C*, invest will be the best choice based on the actual information's and situation.

The figure tells us that the solution must be linear, we can set:

$$F(G,I) = I * f\left(\frac{G}{I}\right) = I * f(C) \tag{114}$$

And the derivatives as:

$$\frac{\partial F}{\partial G}(G,I) = f'(C) \tag{115}$$

$$\frac{\partial F}{\partial I}(G,I) = f(C) - C * f'(C) \tag{116}$$

$$\frac{\partial^2 F}{\partial^2 G}(G,I) = \frac{f''(C)}{I} \tag{117}$$

$$\frac{\partial^2 F}{\partial^2 I}(G,I) = C^2 * \frac{f''(C)}{I} \tag{118}$$

$$\frac{\partial^2 F}{\partial G \partial I}(G,I) = -C * \frac{f''(C)}{I} \tag{119}$$

Substituting this in the Bellman's equation, we obtain:

$$\begin{aligned}\frac{1}{2} * \Bigg[\frac{f''(C)}{I}(\sigma_E^2 * \overline{PR}^2 * E^2) + C^2 * \frac{f''(C)}{I}(\sigma_I^2 * I^2) \\ + 2\left(-C * \frac{f''(C)}{I}\right)(\sigma_E * \sigma_I * \overline{PR} * E * I * \rho_{G;I})\Bigg] + (r - \delta_E) \\ * G * f^{'}(C) + (r - \delta_I) * I * \left(f(C) - C * f^{'}(C)\right) - r * F \\ = 0\end{aligned} \tag{120}$$

$$\begin{aligned}\frac{1}{2} * \Bigg[\frac{f''(C)}{I}(\sigma_E^2 * \overline{PR}^2 * E^2) + C^2 * \frac{f''(C)}{I}(\sigma_I^2 * I^2) \\ + 2\left(-C * \frac{f''(C)}{I}\right)(\sigma_E * \sigma_I * \overline{PR} * E * I * \rho_{G;I})\Bigg] + (r - \delta_E) \\ * G * f^{'}(C) + (r - \delta_I) * I * \left(f(C) - C * f^{'}(C)\right) - r * F \\ = 0\end{aligned} \tag{121}$$

[57]
$$\frac{1}{2} * \left[\sigma_E^2 + \sigma_I^2 - 2 * \rho_{G;I} * \sigma_E * \sigma_I * \overline{PR} * E * I\right] * \left(C^2 * f''(C)\right) + [\delta_I - \delta_E\ ] * \left(C * f'(C)\right) - [\delta_I] * f(C) = 0 \quad (122)$$

The value-matching conditions becomes:

$$f(C) = \frac{C}{\delta_E} - 1 \quad (123)$$

The smooth-pasting conditions becomes:

$$f'(C) = \frac{1}{\delta_E} \quad (124)$$

$$f(C) - C * f'(C) = -1 \quad (125)$$

Given these conditions, we can solve the equation:

$$\frac{1}{2} * \left[\sigma_E^2 + \sigma_I^2 - 2 * \rho_{G;I} * \sigma_E * \sigma_I * \overline{PR} * E * I\right] * \left(\beta * (\beta - 1\ )\right) + [\delta_I - \delta_E\ ] * (\beta) - [\delta_I] = 0 \quad (126)$$

$$\frac{1}{2} * \left[\sigma_E^2 + \sigma_I^2 - 2 * \rho_{G;I} * \sigma_E * \sigma_I * \overline{PR} * E * I\right] * \beta^2 + \left[\delta_I - \delta_E - \left(\frac{1}{2} * \left[\sigma_E^2 + \sigma_I^2 + 2 * \rho_{G;I} * \sigma_E * \sigma_I * \overline{PR} * E * I\right]\right)\right] * (\beta) - [\delta_I] = 0 \quad (127)$$

Dividing by σ^2:

$$\frac{1}{2} * [\sigma^2] * \beta^2 + \left[\delta_I - \delta_E - \left(\frac{1}{2} * [\sigma^2]\right)\right] * (\beta) - [\delta_I] = 0 \quad (128)$$

$$\frac{1}{2} * \beta^2 + \left[\frac{(\delta_I - \delta_E)}{\sigma^2} - \left(\frac{1}{2}\right)\right] * (\beta) - \left[\frac{\delta_I}{\sigma^2}\right] = 0 \quad (129)$$

__Roots and solution of the equation:__

Where β is equal to:

[57] The symbol after σ_I^2 should be a "+" but is converted into "-" to respect the rule given by Var(X-Y)

$$\beta = \frac{-b \pm \sqrt{b^2 - 4 * a * c}}{2 * a} \tag{130}$$

$$a = \frac{1}{2} \; ; b = \left[\frac{(\delta_I - \delta_E)}{\sigma^2} - \left(\frac{1}{2}\right)\right] ; \; c = -\left[\frac{\delta_I}{\sigma^2}\right] \tag{131}$$

$$\beta_1 = \frac{-\left[\frac{(\delta_I - \delta_E)}{\sigma^2} - \left(\frac{1}{2}\right)\right] + \sqrt{\left[\frac{(\delta_I - \delta_E)}{\sigma^2} - \left(\frac{1}{2}\right)\right]^2 - 4 * \left(\frac{1}{2} * -\left[\frac{\delta_I}{\sigma^2}\right]\right)}}{2 * \frac{1}{2}} \tag{132}$$

$$\beta_1 = \left[-\frac{(\delta_I - \delta_E)}{\sigma^2} + \left(\frac{1}{2}\right)\right] + \sqrt{\left[\frac{(\delta_I - \delta_E)}{\sigma^2} - \left(\frac{1}{2}\right)\right]^2 + 2 * \left[\frac{\delta_I}{\sigma^2}\right]} \tag{133}$$

$$\boxed{\beta_1 = \left[-\frac{(\delta_I - \delta_E)}{\sigma^2} + \left(\frac{1}{2}\right)\right] + \sqrt{\left[\frac{(\delta_I - \delta_E)}{\sigma^2} - \left(\frac{1}{2}\right)\right]^2 + 2 * \left[\frac{\delta_I}{\sigma^2}\right]}} \tag{134}$$

$$\beta_2 = \left[-\frac{(\delta_I - \delta_E)}{\sigma^2} + \left(\frac{1}{2}\right)\right] - \sqrt{\left[\frac{(\delta_I - \delta_E)}{\sigma^2} - \left(\frac{1}{2}\right)\right]^2 + 2 * \left[\frac{\delta_I}{\sigma^2}\right]} \tag{135}$$

The first root is greater than the second and the system produces a solution only when:

$$If\ \delta_I\ and\ \delta_E\ are\ both > 0\,, \qquad then\ \ \beta_1 > 1$$

If it were not the case, I or G will lose value with the time and the system will become unsolvable. If one of the values are larger than the risk-free interest, the system also becomes unsolvable by mathematical relation.

We find:

$$\boxed{C^* = \frac{G^*}{I^*} = \frac{E^* * \overline{PR}}{I^*} = \frac{\beta_1}{\beta_1 - 1} * \delta_E} \tag{136}$$

$$\boxed{F(G, I) = I * f(C) = I * \left(\frac{C}{\delta_E} - 1\right) = I * \left(\frac{\frac{E * \overline{PR}}{I}}{\delta_E} - 1\right)} \tag{137}$$

<u>Comparison test with the results of the reference research article</u>

Computing the solution with the values:

$\alpha_E = -0.022, \alpha_I = -0.118, \sigma_I = 0.2, \sigma_I = 0.2, \rho_{E;I} = 0.0, r_{freerisk} = 0.03, \rho_{E;M} = 0.8,$

$\rho_{I;M} = -0.8, r_M = 0.09, \sigma_M = 0.2$

An optimal ratio of 1.863 is produced which is in line with the results of the article of McDonald & Siegel.

5. Model 3: Least Square Monte-Carlo (LSM)

As explained before, LSM is a useful tool to value complex real options. Some precisions will be made about this technique to avoid misunderstandings about the results.

In a real option value problem, it's necessary to quantify the expected value of the project, in the LSM model it can be achieved through a linear regression based on the value of the variables at each time. The option is supposed to be a Markov chain where all the information is summarized in the actual value. Using this simple forecasting should lead to an accurate value but the coefficients can be not significant, their values varies with the time and can be significative at 99% on time 50 but not at 95% on time 100. As stated in the paper of Longstaff & Schwartz, it can be explained by an unexpected change in the value of the variables due to the uncertainty and so the significance of the coefficients is less relevant. The risk of not including an important variable is higher than the influence of an insignificant coefficient[58].

A second advantage of LSM is that it requires around 1000 simulation to get a convergence to the solution, it decreases considerably the time of computation.

The weak point of this method is that underestimate the value of the option compared to the analytical solution. It's why some cautious need to be done when the LSM is only the available solution. The simulation considers continuous values where each increment (dE) is summed to the value of time t-1.

a) Variables considered in the model

The values used in this model will be the same than presented in the summary table above. A reminder is provided here for information.

Electricity price (E): stochastic

$$d\,E = \mu_E * E * dt + \sigma_E * E * dW_t \tag{138}$$

Quantity of energy produced (Q): mean by year or month with stochastic increment

$$d\,Q = \bar{\eta} * (\bar{Q} - Q) * dt + \sigma_E * dW_t \tag{139}$$

The solar panel produces in the best conditions of production.

Investment cost (I): stochastic with a decreasing trend

$$d\,I = \alpha_I * I * dt + (\sigma_I * I * dW_2) \tag{140}$$

The cost of replacement of the inverter will be integrated in the initial investment cost for a value of 250€ actualized at the rate r for 10 years.

[58] Testing of the model provides better results when all the regressions were used, it has also been used by other researches on the method.

The periodicity of the model is monthly with a continuous compounding (a year is considered here to be 12 months of 30 days). Values expressed in yearly format are converted into monthly format.[59]

General information:		
Life of the installation (guarantee)	20 years	
Maturity of the option	10 years	
Power of the installation	3 kWc	
Electricity price variable (E):		range [100€ ; 700€]
Initial value:	220, 260 and 300	
Trend:	2%, 3%, 4% and 7%/year	
Volatility:	0,2/year	
Electricity produced variable (Q):		range [20 ; 200]
Initial value	104,96	
$\bar{Q}$	104,96	
$\bar{\eta}$	3,1033	
$\hat{\sigma}$	126,2737	
Loss factor	0,5%/year	
Investment cost (I):		range [500€ ; 7000€]
Initial value:	6.500,00€	
Trend:	-11,5%/year	
Volatility:	0,075/year	
Inverter cost:	$\frac{250}{(1+r)^{10}}$	Summed with initial I
Maintenance cost:	0,75% * I	
Walloon Prosumer Tax (PRO_WL):		
Initial value:	78€ /kWe	Valued as (1-37,76%) of the kWc
Trend:	1%/year	
Auto consumption factor:	37,76%	
Flemish Prosumer Tax (PRO_FL):		
Initial value:	110€ /kW of kVA	Valued as initial kWc
Trend:	1%/year	
Brussels Certificat Vert (CV):		
Initial value:	96€/CV	
Trend:	0,3%/year	
Volatility:	0,03/year	range [65€ ; 100€]
CV number	3 CV/ 1.000 kWh	
Interest rate (r)	0,5%; 1%; 3% and 5%/year	

[59] Interest rate monthly (R_int_monthly) = $\sqrt[12]{(1+R_{int_{yearly}})}-1$

b) Scenarios

Multiple scenarios are considered in the model based on the different regimes applicable in the regions of Belgium in 2019:

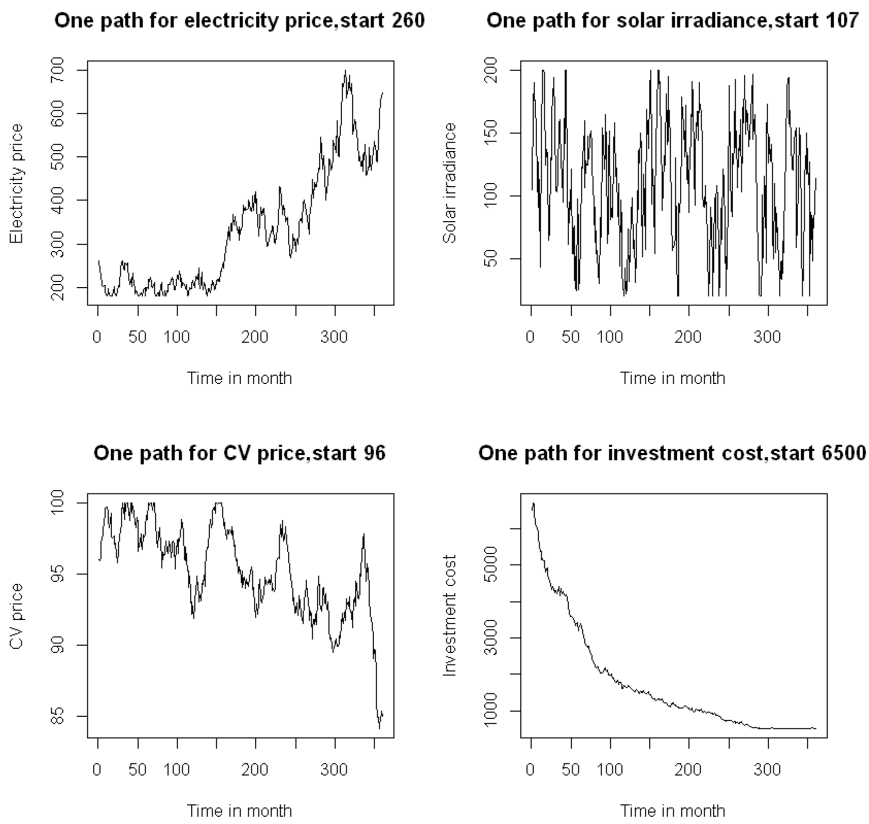
- A1: Situation of a solar panel without region specificities (basis)
- A2: Situation of a solar panel in Brussels region with a CV system
- A3: Situation of a solar panel in Walloon region with the Prosumer tax (PRO_WL)
- A4: Situation of a solar panel in Flemish region with a Prosumer tax (PRO_FL)

c) How the algorithm works

The algorithm runs with the inputs of the variable's summary table, default values are set in the case where no inputs are provided. The output takes the form of a data frame that stores all the results from the paths of each simulation of each variable, cash-flows, NPV and option value (some treatments are necessary to show the results). Data's can easily be plotted to obtain a visual form of the results. The word "activation" means here "use the call to invest in the project".

<u>Process:</u>

1) Based on the inputs of the model, a set of paths for all the stochastic variables is generated. One example is provided on the next graph for Electricity price, Solar irradiance, CV price and Investment cost (one period means one month).



2) Cash-flows are computed through the valuation formulas of the inputs.

❖ A1: Situation of a solar panel without region specificities (basis)

Elec_price (in mWh) * Solar_irradiance (in kWh) *3kWc *0.86(conversion factor of the energy) /1000 – Maintenance_cost (141)

❖ A2: Situation of a solar panel in Brussels region with a CV system

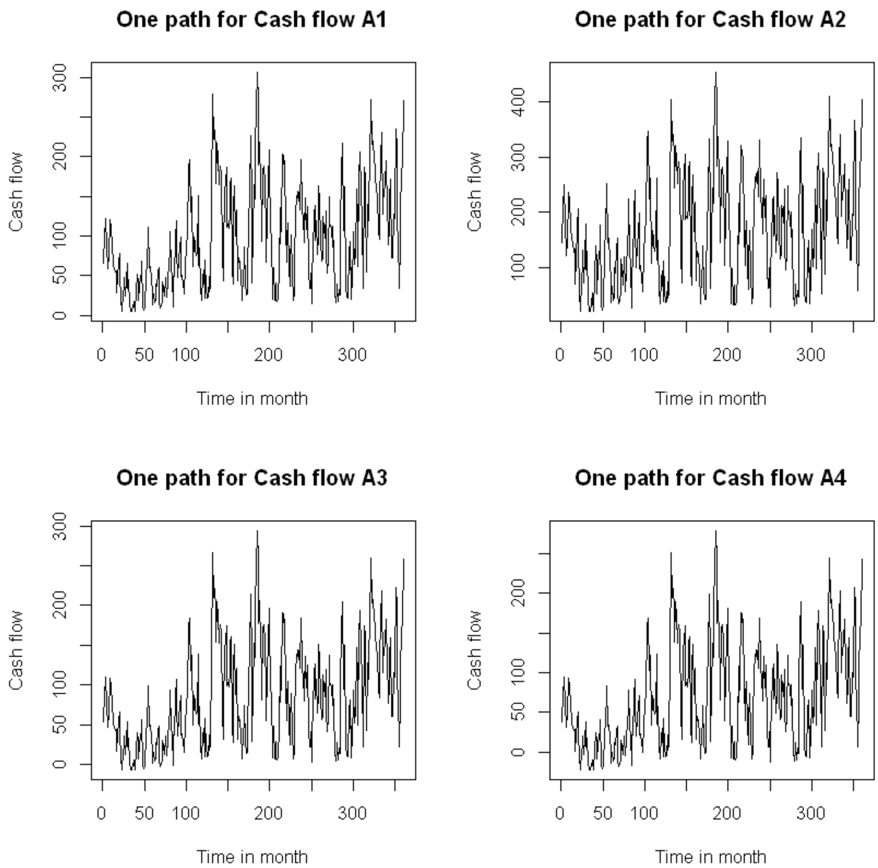
Elec_price (in mWh) *Solar_irradiance (in kWh) *3kWc *0.86(conversion factor of the energy) /1000 – Maintenance_cost + 3* CV_price *Solar_irradiance (in kWh) *3kWc *0.86(conversion factor of the energy) /1000 (142)

❖ A3: Situation of a solar panel in Walloon region with the Prosumer tax (PRO_WL)

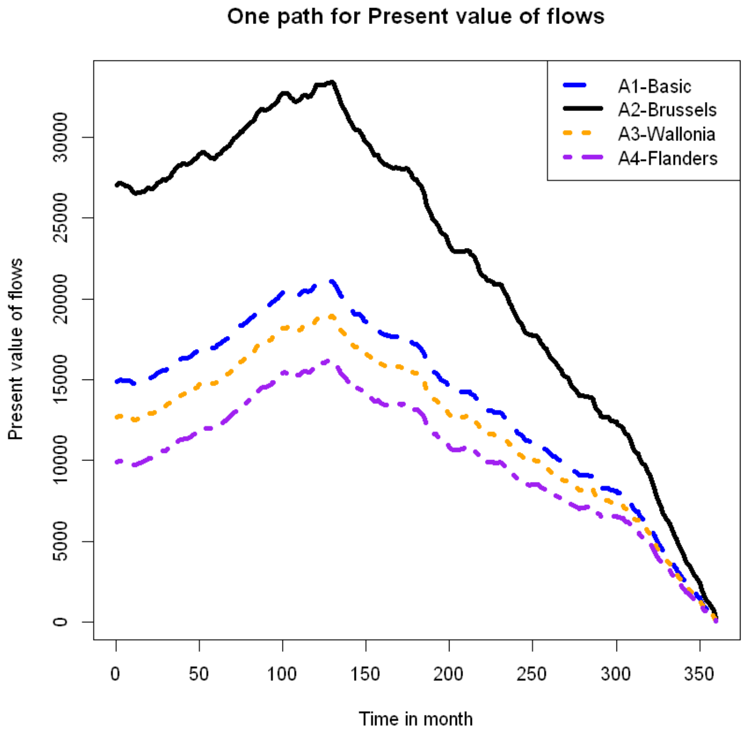
Elec_price (in mWh) * Solar_irradiance (in kWh) *3kWc *0.86(conversion factor of the energy) /1000 – Maintenance_cost - PRO_WL *3kWc * 1/12 (monthly payment) (143)

❖ A4: Situation of a solar panel in Flemish region with a Prosumer tax (PRO_FL)

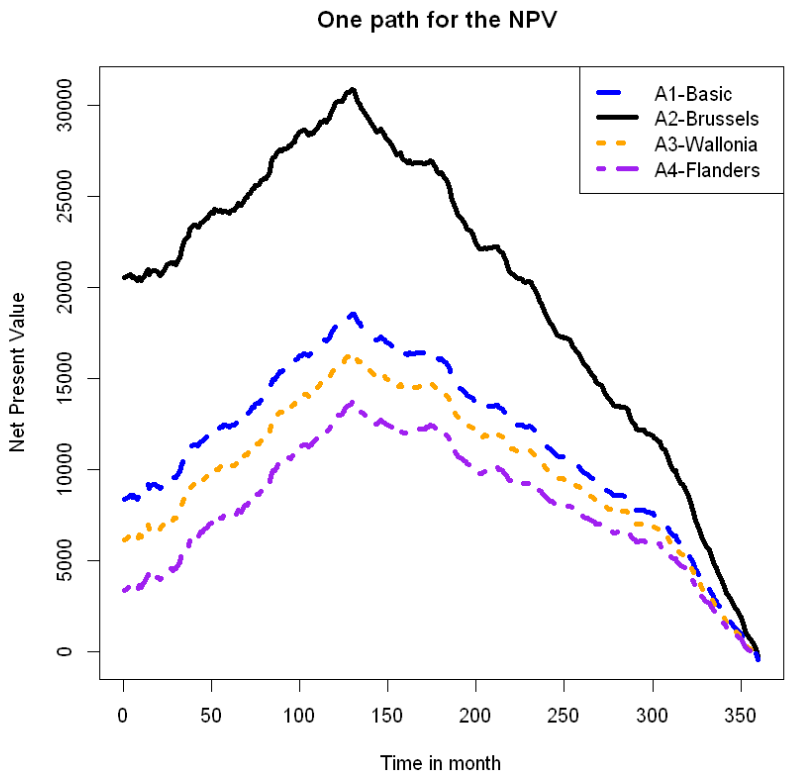
Elec_price (in mWh) * Solar_irradiance (in kWh) *3kWc *0.86(conversion factor of the energy) /1000 – Maintenance_cost - PRO_FL *3kWc * 1/12 (monthly payment) (144)



3) Present values of those cash-flows are calculated but it doesn't include the investment cost. It's just more convenient to compute the value of the optimal ratio with the next step.



4) Net present value is determined by the difference between Present values and Investment cost (maximum at the end of option maturity)



5) Termination value of the ROV which is a kind of 0 strike American option, is determined based on the following rule:

$$Real\ Option\ Value = \max(NPV; 0) \qquad (145)$$

6) Implementation of the Least Square Monte Carlo (LSM): it follows a roll-back equilibrium (the process is recursive in time) and at each time t values are computed as:

1. Determination of the actual value based on the NPV_{t+1} actualized once at the interest rate (r which is an input)

2. Determination of the continuation value (value of the option not activated): it's based on a linear regression on the main variables and change for each scenario. The results of each regression are used to predict the real option value based on the information's at time t of the stochastic variables (a basic case of forecasting with the most important values)[60]:

$$A1: \ Actualized_{value} = \alpha + B_1 * E + B_2 * E^2 + B_3 * Q + B_4 * Q^2 + B_5 * I + B_6 * I^2 + B_7 * (E * Q) + B_8 * (E * I) + B_9 * (Q * I)$$

$$A2: \ Actualized_{value} = \alpha + B_1 * E + B_2 * E^2 + B_3 * Q + B_4 * Q^2 + B_5 * I + B_6 * I^2 + B_7 * (E * Q) + B_8 * (E * I) + B_9 * (Q * I) + B_{10} * CV + B_{11} * CV^2 + B_{12} * (E * CV) + B_{13} * (Q * CV) + B_{14} * (I * CV)$$

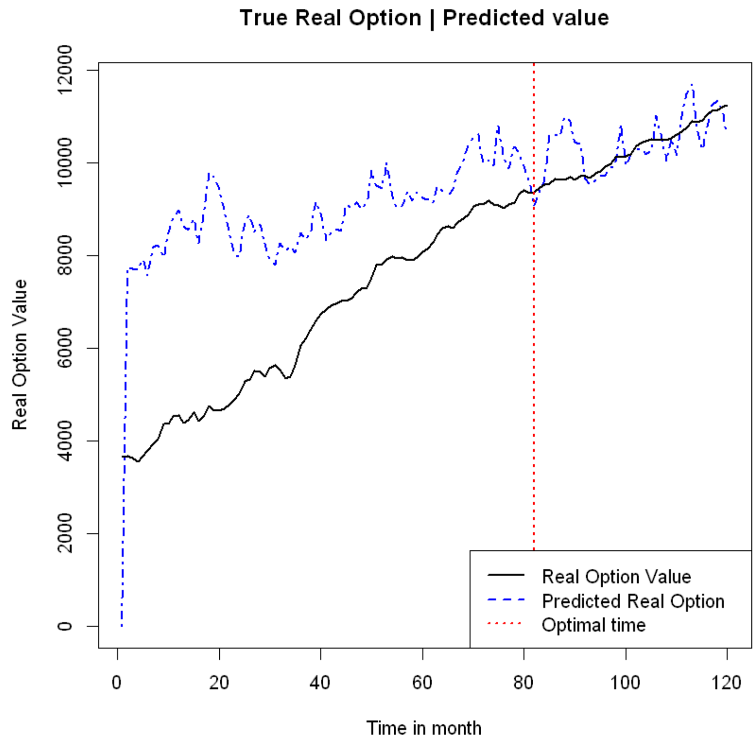
$$A3: \ Actualized_{value} = \alpha + B_1 * E + B_2 * E^2 + B_3 * Q + B_4 * Q^2 + B_5 * I + B_6 * I^2 + B_7 * (E * Q) + B_8 * (E * I) + B_9 * (Q * I) + PRO\_WL$$

$$A4: \ Actualized_{value} = \alpha + B_1 * E + B_2 * E^2 + B_3 * Q + B_4 * Q^2 + B_5 * I + B_6 * I^2 + B_7 * (E * Q) + B_8 * (E * I) + B_9 * (Q * I) + PRO\_FL$$

3. Determination of activation value is given by the rule: $\max(NPV_t; 0)$

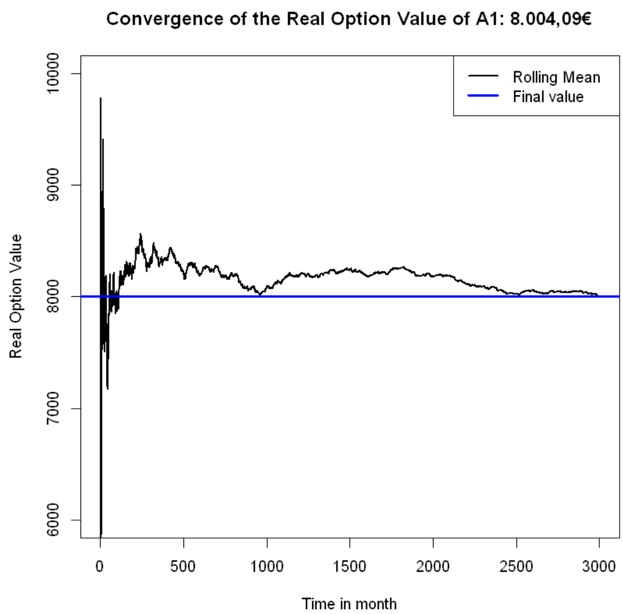
4. The maximum between the activation of the option and the continuation value is chosen. When the activation is the best choice (true value > predicted value), a dummy takes the value 1 to determine at which time it's optimal to invest. On the next graph, the optimal time is highlighted with a red vertical line (in this case at time 82 or 8,6 years for this path).

[60] After several test the basis function provides better results even if some coefficients are not significant, the use of the complete model will be preferred for this reason. The option value slightly lower with a reduced model but miss more often the maximum NPV of the path.

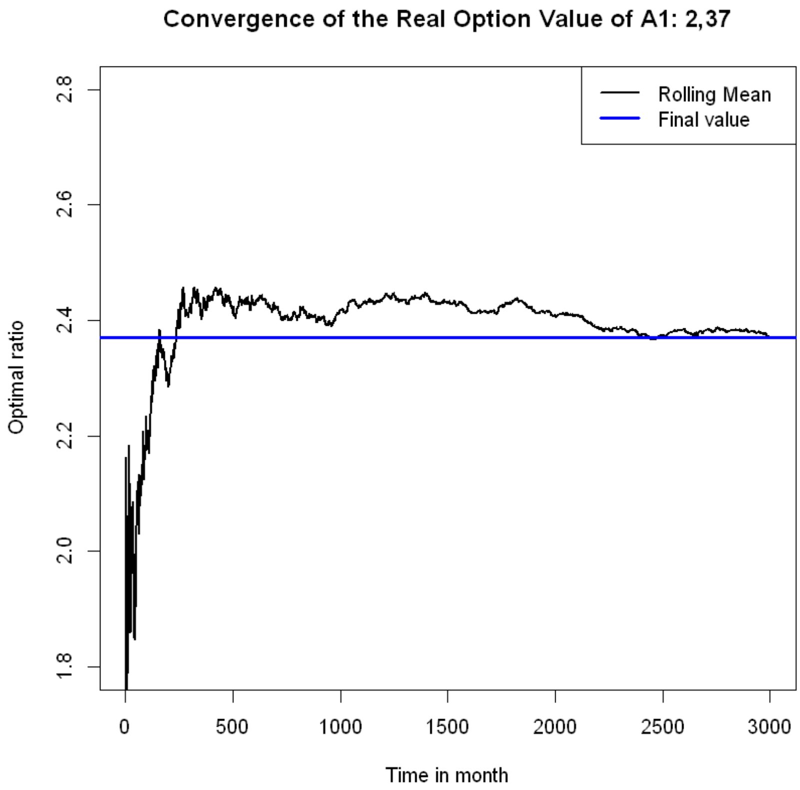


6) The previous steps are done for each simulation and give a table of the NPV at the first-time t of the activation (best opportunity of activation).

7) The mean of all the values of step 6 is computed and gives the Real Option Value at time 0. The convergence is measured by the contribution to the mean of each simulation. A number of simulations about 1200 is generally enough to obtain a convergence of the option price. More simulations induce more computation times and don't bring additional precisions to the calculus.



8) Based on the optimal activation time of each simulation, the ratio Flows/Investment costs is determined. The mean and the convergence are calculated in the same way than the mean of option value in step 7.



6. Comparison of the results

a) Results

Model 1 & 2: Results with the basic scenario

The 2 first model will be tested together as they provide both an analytical solution. For model 1 and 2, the value G is determined by:

$$G = \sum_{i=1}^{20}(E_t * \overline{PR} * 12\,months * 3kWc * 0{,}86\,(conversion\,factor)/1000) * e^{-r*t} \quad (146)$$

Where G follows:

$$d\,G = \alpha_E * E * \overline{PR} * dt + \sigma_E * \overline{PR} * E * dW_t \quad (147)$$

With:

- Electricity price – trend: 3%/year
- Electricity price – volatility: 0,2/year
- Electricity price – start: 260 €/ mWh
- Production of electricity ($\overline{PR}$): 104.96 kWh/month
- Rate of interest – risk free: 3%/year
- Rate of interest – market: $((1 + 0.3928\%)^{12}) - 1$/year
- Rate of interest – market volatility: 0,018/year
- Correlation – Electricity and Market: -0,04
- Investment cost: 6.500€
- Investment cost – trend: -11,5%/year
- Investment cost – volatility: 0,075/year
- Correlation – Electricity and Market: 0,32

Model 1

Applying formulas n°77 to 79, it gives an analytical solution approximated for a solar panel installation for 20 years. A slight difference appears as the installation doesn't live infinitely, it's why formula n°81 is not used here.

The option value (F) is given by:

$$F = A_1 * G^{\beta_1} + A_2 * G^{\beta_2}$$

Where A_2 is equal to 0,

Results			
Optimal ratio	3,47	**G***	22.602€
A1	0,01244	**A2**	Fixed at 0
Beta 1	1,4036	**Beta 2**	Fixed at 0

The actual value of G is 12.569,77€. The real option value is in this case:

$$A_1 * G^{\beta} = 0{,}01244 * 12.569{,}77^{1{,}4036} = 7.057{,}26\ € \quad (148)$$

This value is positive and means that it should be better to wait. It can be explained by the value of G, it's lower than the optimal level G* (22.602€) and it's not optimal to invest with the actual information's and situation. Using the optimal ratio information, 12.569,77€ is lower than 3.47*6.500€ (investment cost). The same conclusion applies here.
This cost should play a key role in the option value as the trend is negative(-11,5%/year). This specificity is not considered with the model 1 as investment cost is assumed fixed, it's why no additional tests are executed with this first approach.

Model 2
Based on the formula n°136, the model gives:

	Results
Optimal Ratio	-23,66
Beta 1	0,9594
Delta E	-0,8%
Delta I	16,92%

No solution can be obtained from this model because δ_E (delta E, or a measure of the dividend rate of the Electricity price) is negative. Additionally, δ_I (delta E, or a measure of the dividend rate of the Investment cost price) is larger than the interest rate. No convergence could happen with those parameters values. It's equivalent to actualize on an infinite horizon with a negative rate of interest, the value can only increase with the time. Trying to compute the real option value with G (12.569,77€), it gives:

$$W(G,I) = I * \left(\frac{\frac{G}{I}}{\delta_E} - 1\right) = 6500 * \left(\frac{\frac{12.569{,}77}{6500}}{-0{,}8\%} - 1\right) = -1.577.721{,}25 \quad (149)$$

Trying to compute the real option value directly with profit flow:
$E_t * \overline{PR} * 12\ months * 3kWc * 0{,}86\ (conversion\ factor)/1000$, it gives:

$$W(G,I) = I * \left(\frac{\frac{G}{I}}{\delta_E} - 1\right) = 6500 * \left(\frac{\frac{260 * 104.96 * 12 * 3 * \frac{0.86}{1000}}{6500}}{-0{,}8\%} - 1\right) \quad (150)$$
$$= -112.110{,}75€$$

Those results are a special case, the Real Option Value is negative (with the formula) and means that it should be optimal to invest now. The true option value is 0, this relation comes from the fact that a call can never be negative and has a minimum value of 0. The optimal ratio becomes irrelevant in this case.

As the 2 analytical model are not able to provide a normal solution, their results will not be analyzed more in details for this reason. The investment problem will now be tested with a more flexible model which is the LSM model.

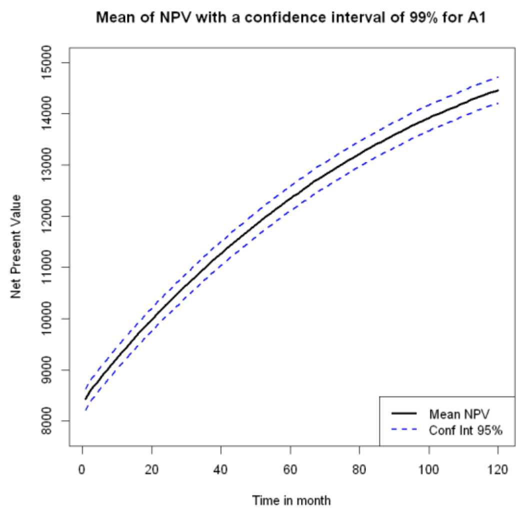
Model 3: LSM Simulation

In a first time, results will be analyzed to identify the relation between the inputs. The model has been tested (3000 simulations) with all the inputs of the summary table. The scenario considers here:

- Rate of interest: 3%
- Electricity price – trend: 3%

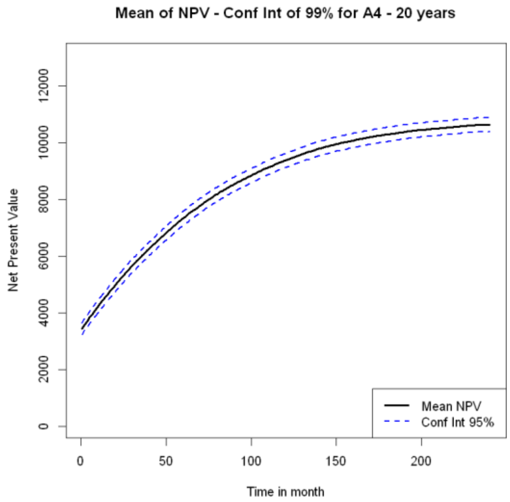
Electricity price - Real Option Value			
Start	**220**	**260**	**300**
A1	7.324,29 €	8.004,10 €	9.546,24 €
A2	17.283,48 €	18.507,17 €	20.115,81 €
A3	5.748,73 €	6.440,33 €	7.873,79 €
A4	4.009,59 €	4.486,25 €	5.993,04 €
Electricity price - Optimal ratio			
Start	**220**	**260**	**300**
A1	2,24	2,37	2,61
A2	4,29	4,49	4,71
A3	1,90	2,04	2,27
A4	1,51	1,62	1,87

The results give a positive option value and tend to show that it increases with the initial price of electricity. It should mean waiting before investing but the relation with the initial price is not logical. One explication can be the fact that the maturity of the option is only 10 years, if the investment cost decreases in the time and price of electricity increase, it becomes more and more profitable to invest and waiting could bring a higher NPV until the end of the option life (10 years) where the value falls to 0. It can be proved with the next graph that represents the mean of the different NPV's along the generated paths. This is a growing function, NPV will rise with the time and remains positive within a confidence interval of 99% between 8.219,49€ and 8.628,42€ (mean: 8.423,96€; standard deviation: 4.357,61; 3000 simulations). The local maximum of the function only appears at the end of the option maturity. The option value gives the difference, actualized at rate r, between the NPV at the end (around 14.500€) of the option maturity and the NPV at time 0 (around 8.500€). Capitalizing the difference on 10 years (r =3%), it gives a value around 8.000€. It's the same than the real option value which comforts the explication.

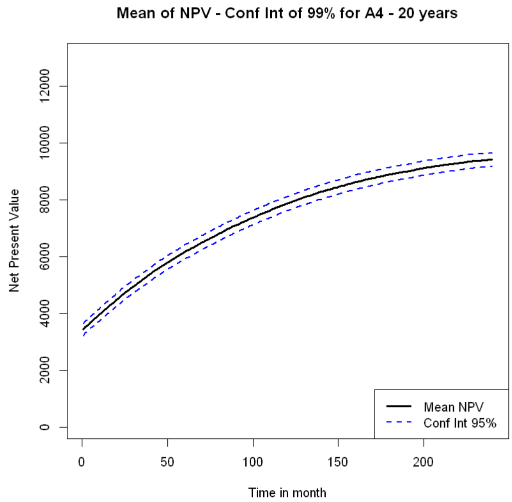


Some additional tests have also been conducted with an option maturity of 20 years on the solar panel in Flemish region (lowest profitability) and with a maturity of 20 years with an investment cost trend of -6%/year. The same conclusion applies here: real option has a growing positive value with the time.

Flemish region (A4)
Option maturity: 20 years
Investment cost trend: -11.6%/year



Flemish region (A4)
Option maturity: 20 years
Investment cost trend: -6%/year



An investor could have a higher profitability by waiting but he can wait a long time (or forever) as the investment cost decreases rapidly. It confirms the results of the model 2, investing now could be treated as an optimal choice even if the real option is positive. It can be considered that's value will be eternal positive[61] as waiting a very long time (20 years) will always paying off compared to an immediate investment.

On the other side, optimal ratio of investment provides every time a value bigger than 1. It confirms the investment theory as the classic rule without uncertainty is to invest when NPV=0 or when the optimal ratio is at least equal to 1 (as explained in the theorical part). In presence of uncertainty, it should be bigger than 1 as uncertainty induces a cost of risk. The results are the lowest for the Flemish case (1,62 for a start price of 260€), it can be explained by the Prosumer tax that reduces the profits and requires to invest more rapidly than in the other regions as the pay-back time is longer (in number of years), in comparison Brussels region has the highest value (4,49 for a start price of 260€). The difference can be explained by the presence of the CV regime (better to wait an increase of their price) and the absence of tax, waiting generate additional profits as the growing trend of electricity price is reinforced by CV selling. The reference book of Dixit&Pindyck [8] shown with a mathematical demonstration that a good approximation of the optimal ratio should be near of 2, which is the case for A1 and A3.

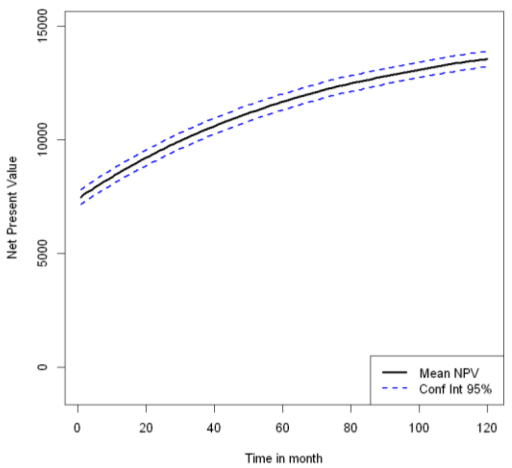
[61] Eternal means here a time horizon bigger than 20 years, the option will lose its value when investment cost will be stable as the electricity price but it will not happen with this time horizon.

To be sure that investing now is an optimal choice, the return of the investment must be compared with the return of the stock market for example. It can be the situation of an investor who hesitate to spend money between an ETF (on Eurostoxx 50 or on S&P500) or with a solar panel investment. To compare both profitability, ETF return will be approximated by historical rate of profit and solar panel will be valued with the IRR method (for a given interest rate, the average of NPV's (time 0) should be close to 0). The reasons are that the investments in an ETF will generate returns (sometimes positive, other times positive) but no sunk appears. It's different for the case of a solar panel, most of the returns will be positive (depend of the solar irradiance which is constant on a long-time horizon), but a sunk cost is spent at the initial time. The impact of this out-flows generate difficulties to compare their total profitability.

Tests will be now divided by region[62]:

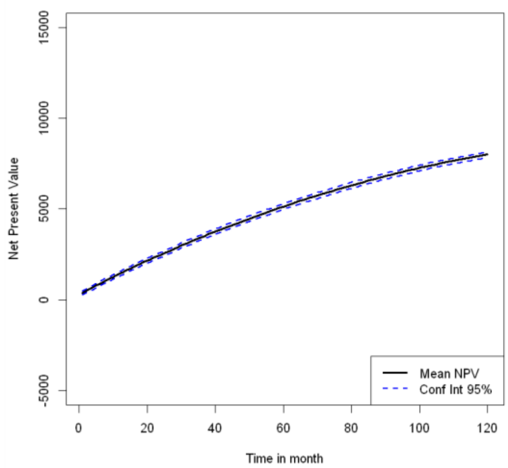
a) Flemish region (A4):
The historical trend of electricity price is about 6% and the price on the 01/01/2019 is 300€/mWh, the simulation with an annual interest rate of 3% gives:

Flemish region (A4)
Trend electricity: 6%
Initial price: 300€
Interest rate: 3%



Real option Value: 8.079,72
Optimal ratio: 2,40
Mean NPV time 0: 7.466,35
Sd NPV time 0: $4.834{,}07/\sqrt{1500}$

Flemish region (A4)
Trend electricity: 6%
Initial price: 300€
Interest rate: 12%



Real option Value: 1.466,06
Optimal ratio: 1,28
Mean NPV time 0: 377,85
Sd NPV time 0: $1748{,}75/\sqrt{1500}$

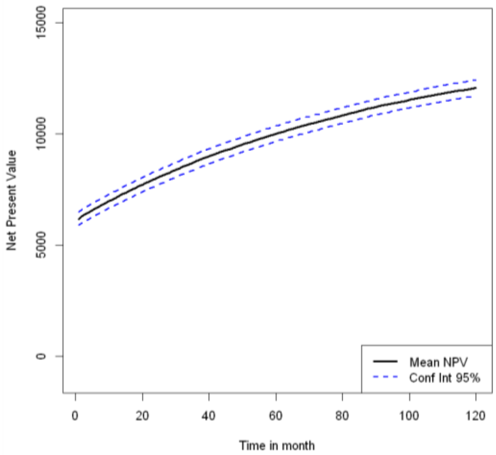
The IRR in the Flemish region is about 12%/year

[62] Based on 1500 simulations, horizon of 10 years and show on 20 years for the IRR determination

b) Walloon region (A3):

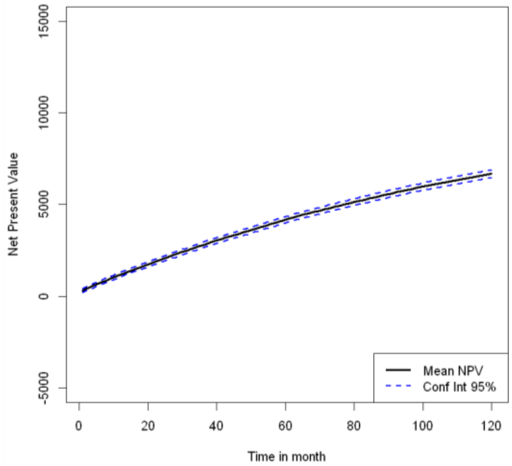
The historical trend of electricity price is about 3% and the price on the 01/01/2019 is 260€/mWh, the simulation with an annual interest rate of 3% gives:

Walloon region (A3)
Trend electricity: 3%
Initial price: 260€
Interest rate: 3%



Real option Value:	1.392,14
Optimal ratio:	1,10
Mean NPV time 0:	244,39
Sd NPV time 0:	$4.479,09/\sqrt{1500}$

Walloon region (A3)
Trend electricity: 3%
Initial price: 260€
Interest rate: 11%



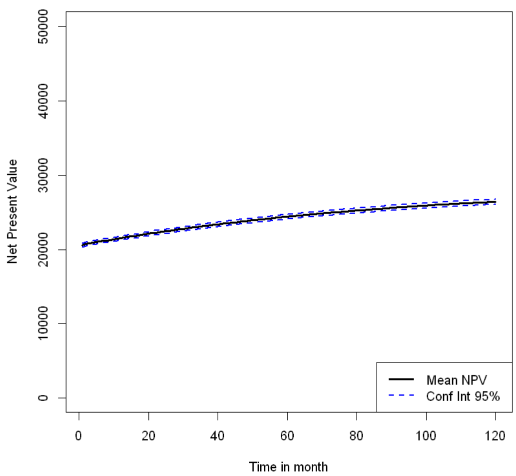
Real option Value:	1.347,17
Optimal ratio:	1,22
Mean NPV time 0:	318,18
Sd NPV time 0:	$1.685,65/\sqrt{1500}$

The IRR in the Walloon region is about 11%/year

c) Brussels region (A3):

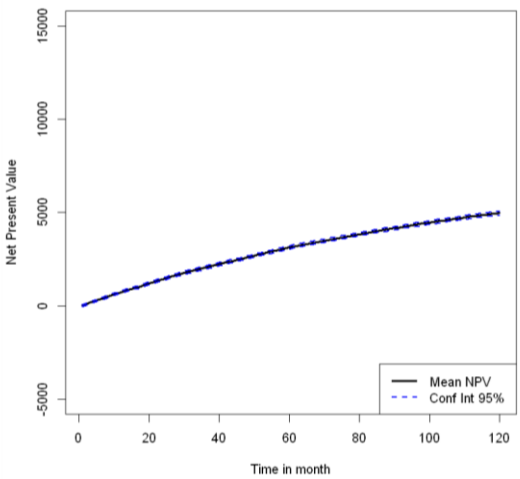
The historical trend of electricity price is about 2% and the price on the 01/01/2019 is 220€/mWh, the simulation with an annual interest rate of 3% gives:

Brussels region (A2)
Trend electricity: 3%
Initial price: 220€
Interest rate: 3%



Real option Value:	16.371,96
Optimal ratio:	4,11
Mean NPV time 0:	20.555,54
Sd NPV time 0:	$4226/\sqrt{1500}$

Brussels region (A2)
Trend electricity: 2%
Initial price: 220€
Interest rate: 28%



Real option Value:	551,99
Optimal ratio:	1,01
Mean NPV time 0:	17,25
Sd NPV time 0:	$828,02/\sqrt{1500}$

The IRR in the Brussels region is about 28%/year

Those results can be found as exceptional and will be compared to the historical profitability. As Wallonia applied a CV system and a direct Subsidy, this case is perfect to test the value obtained for Brussels (A2).

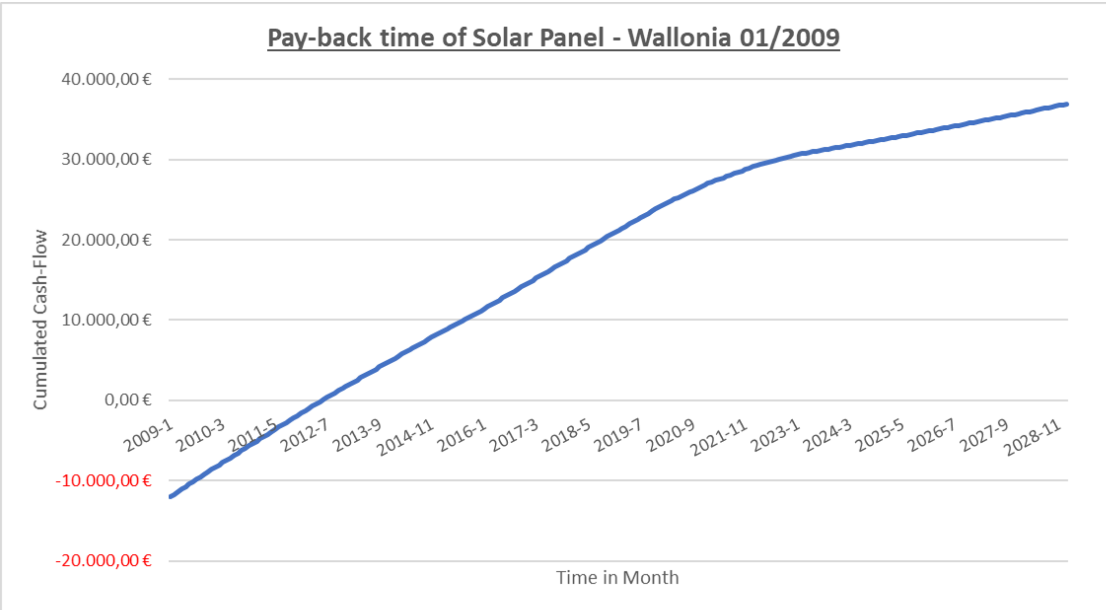
b) Historical profitability level of a solar panel in Wallonia

Wallonia will be used as a benchmark of profitability, it's the region where most of the data are available and provides interesting results with the multiple subsidy regime.

CV in Wallonia
Considering an installation in 2009 in Wallonia (CV regime) with[63]:

- A life installation of 20 years
- an initial cost of 12.000€[64]
- a constant solar irradiance of 104.79
- The time-series data of Walloon CV from 01/2009 - 12/2018, after valued at the minimal price 65€
- Trend of electricity price of 3% based on the data of the graph from EDF Luminus

The NPV (01/2009) for this investment is 27.278,08€, the IRR is 31%/year and has a pay back-time of 41months (3,4 years).

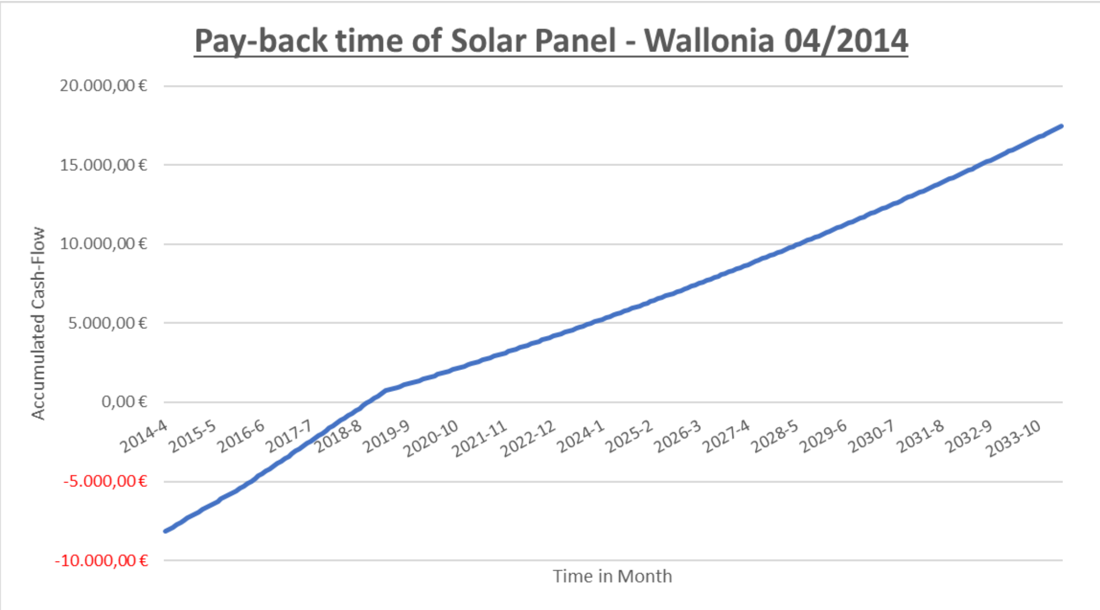


[63] Based on data for a 10 kWc by simplification
[64] https://www.engie-electrabel.be/fr/blog/solutions-pour-la-maison/levolution-des-panneaux-photovoltaiques-depuis-10-ans, consulted on 21/05/2019

Direct subsidy in Wallonia
Considering an installation in 2014 in Wallonia (Qualiwatt subsidy) with:

- A life installation of 20 years
- an initial cost of 8.300€ (data of CWAPE)
- a constant solar irradiance of 104.79
- Trend of electricity price of 3% based on the data of the graph from EDF Luminus

The NPV (04/2014) for this investment is 11.492,40€, the IRR is 18,4%/year and a pay back-time of 56 months (4,7 years).



The results of the simulation are totally plausible because the past profitability was much higher. The CV system provides twice profits than the basis situation.

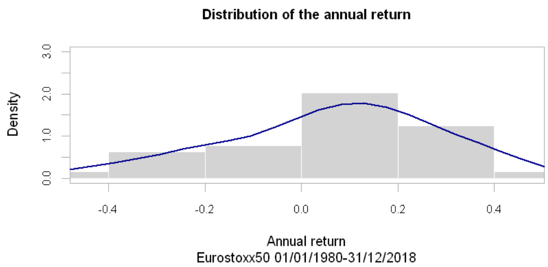
For the ETF, the distribution of the returns and the evolution of the stock price[65] is:

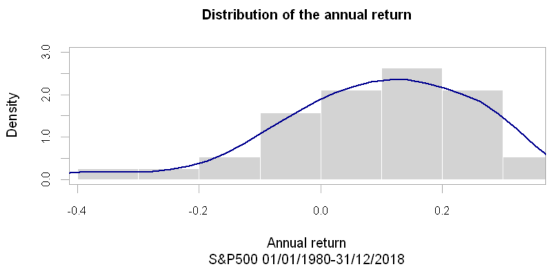
Eurostoxx50

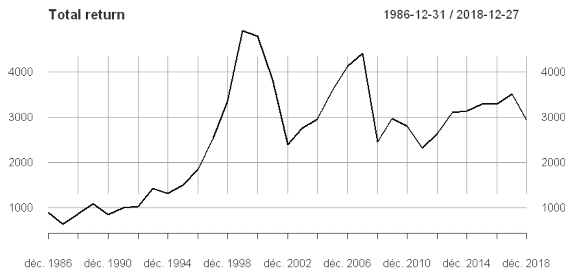
Mean:	6,28%
Standard Deviation:	0,2227
Kurtosis:	-0,38
Skewness:	-0,3968
Var 99%:	-32%
Expected Shortfall 99%:	-40,6%

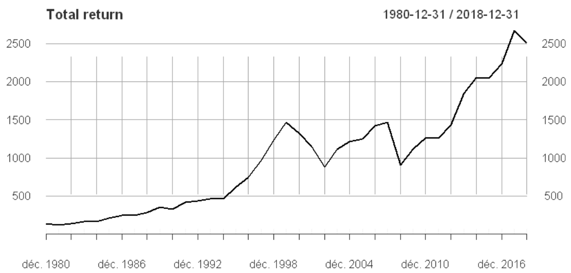
S&P500

Mean:	9,2%
Standard Deviation:	0,1586
Kurtosis:	0,64
Skewness:	-0,7731
Var 99%:	-19,57%
Expected Shortfall 99%:	-28,56%









The distribution of the returns of the 2 indices show a mean of 6,28 and 9,2%, it involves also the presence of downside risk which is not negligible. The Expected shortfall is equal to -40,6% and to -28,56% at the significant rate of 99%. When economics condition goes well, the return is high but when a crisis start, the return can fall drastically. Those bad effects are canceled with the option and the installation itself generate most of the time positive returns, negative returns are very exceptional and very small to be neglected.

For the 4 scenarios the returns are:

Distribution of Solar panel return		
Scenario	Mean of annual return	Sd of annual return
A1 - Basis	19,84%	0,005636
A2 - Brussels	30,45%	0,003291
A3 - Wallonia	12,68%	0,003741
A4 - Flanders	13,89%	0,005543

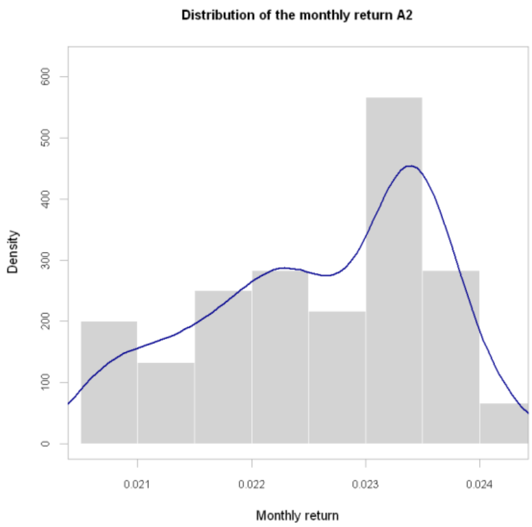
Distribution of Stock exchange return		
Indice	Mean of annual return	Sd of annual return
Eurostoxx 50	6,28%	0,222700
S&P 500	9,20%	0,158600

The following graphs show the details about the results obtained for the 4 scenarios.

[65] Data's adjusted of dividends and extracted from yahoo finance on 21/05/2019

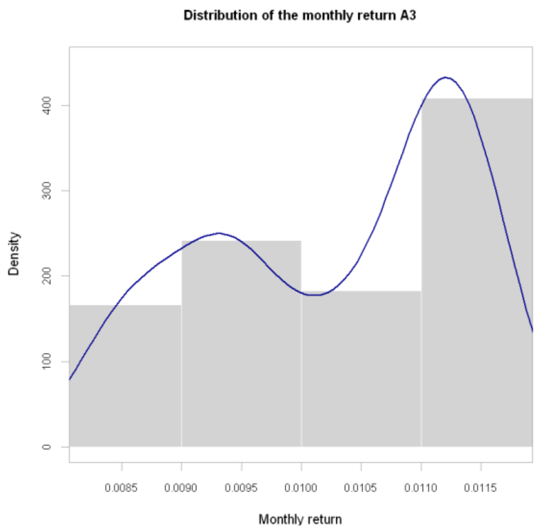
Brussels Region A2

Mean:	2,24%/month
Standard Deviation:	0,00095/month
Kurtosis:	-0,86
Skewness:	-0,29
Var 99%:	> 0%
Expected Shortfall 99%:	> 0%



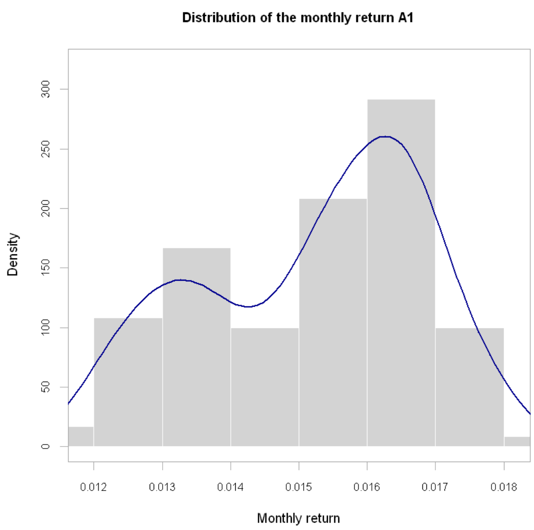
Walloon Region A3

Mean:	1%/month
Standard Deviation:	0,00108/month
Kurtosis:	-1,30
Skewness:	-0,3763
Var 99%:	> 0%
Expected Shortfall 99%:	> 0%



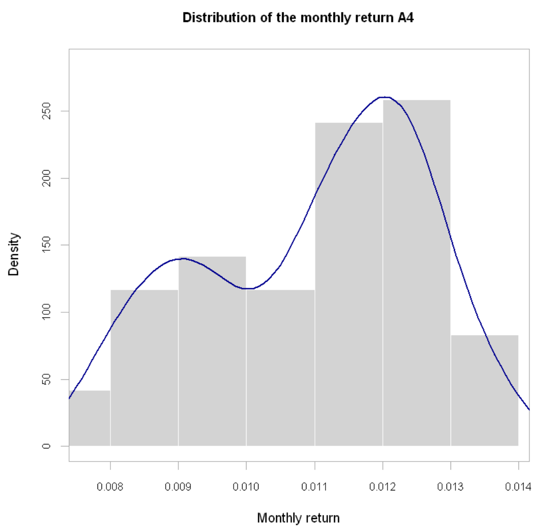
Basis situation A1

Mean:	1,52%/month
Standard Deviation:	0,001627/month
Kurtosis:	-0,9768
Skewness:	-0,3917
Var 99%:	> 0%
Expected Shortfall 99%:	> 0%



Flanders Region A4

Mean:	1,09%/month
Standard Deviation:	0,0016/month
Kurtosis:	-0,9768
Skewness:	-0,3917
Var 99%:	> 0%
Expected Shortfall 99%:	> 0%



c) Sensitivity analysis based on the LSM model

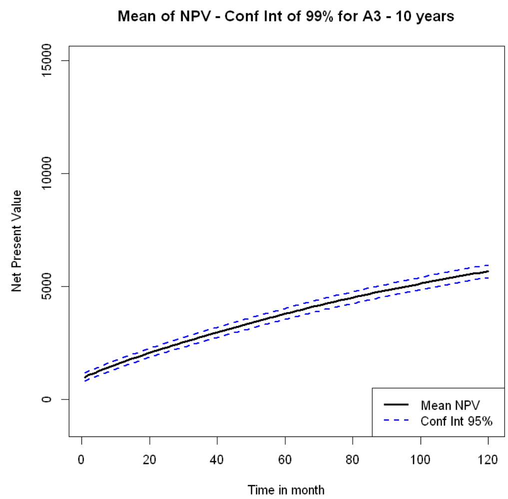
Analyzing the influence of interest rate with value of 0,5%; 3% and 5%, it gives:

Electricity price - Real Option Value			
R_int	0,5%	3,0%	5,0%
A1	11.848,02 €	8.004,10 €	5.936,19 €
A2	25.405,49 €	18.507,17 €	14.744,72 €
A3	9.829,02 €	6.440,33 €	4.610,20 €
A4	7.294,58 €	4.486,25 €	3.000,37 €
Electricity price - Optimal ratio			
R_int	0,5%	3,0%	5,0%
A1	3,02	2,37	2,02
A2	5,66	4,49	3,83
A3	2,62	2,04	1,73
A4	2,11	1,62	1,35

When the rate of interest increases, it contributes to reduce the value of the option which is not in line with the classical investment theory. Some cautious about the results on the option value is necessary as explained before, it means here that a higher interest reduces the NPV and by the same way the option value which is the NPV after 10 years. This conclusion is more logical from a theorical point of view.

A kind of stress test has been used where it considers bad investments conditions:
Investment cost trend: -5,5%/year; Prosumer tax in Wallonia: 120 instead of 78; Electricity trend of 2%/year; Electricity initial price of 180; Interest rate of 5%/year.

For Wallonia, the mean of NPV at time 0 is still positive in those conditions with a value of 997,91€ (sd: 2.667,87€). Profitability is resistant to hard context.



d) Impact of the results

Impact on people

As shown with the results, the investment in solar panel is already profitable, some could wait to increase the total return, but it can mean waiting indefinitely. This investment allows by the same way to be protected of the increase of electricity price in the future which is a kind of additional insurance provided to the owner of a solar panel. One of the problems with this view is the necesity to have a minimal amount of money to pay the investment cost (with the salary or by savings) or to be able to contract a loan to finance this cost. It doesn't generate difficulties for middle and high incomes (many banks provide attractive rate of 3% for green investment) but for low incomes, it represents a disadvantage if they have already reached a maximal level of indebtment (max 1/3 of the monthly income[66] in Belgium) or if the banks don't agree to grant this credit.

Energy cost on its side represents a fixed charge for the budget of a household, people with high incomes will not spend 2 times more money into the heating of their house than a low-income household. It creates inequality as low incomes are forced to pay the growing electricity price while the high income can invest and earn high return with a protection on the energy price increase. Some measures of the state or the regions have been put in place to avoid this problem with, for example in Wallonia, a loan with a 0% rate[67]. Effectively, it contributes to facilitate the access to solar panel investment to a larger number of households, but a such installation doesn't cover totally the electricity expenses as people are now incited to fit the installation size just below their consumption level to pay less Prosumer tax.

Contracting a loan represents an additional monthly cost for the borrowers. It's only when the regular payments will be equal or lower to the energy savings that low-income people could do a tradeoff to invest in this technology.

$$\begin{gathered}\text{Tradeoff strategy: Invest in solar panel and contract a 0\% rate} \\ \text{loan when:} \\ \text{Regular monthly payments} \leq \text{Monthly energy savings}\end{gathered} \tag{151}$$

With the inputs of the basis hypothesis: a loan of 6.500€ with a rate of 0% for 10 years, costs a monthly amount of 54,17€. The solar irradiance must be at least of 86 kWh/m² to cover the loan cost. Excepted in winter, this level is achieved most of the time. People even with the low income should invest to protect of the electricity price.

A barrier to an investment behavior in solar panel is simply the necessity to have a roof, it means to have a house or an apartment which can be difficult even for middle income as young peoples. If they rent something, the landlord doesn't have an interest to install solar panel as the energy consumptions are at charged of the tenants.

Another problem is the investment horizon, the life of a solar panel is guaranted for 20 years and could live 10 years more. It requires to avoid any move for 30 years which doesn't

[66] https://www.wikifin.be/fr/thematiques/emprunter/credit-hypothecaire/comment-bien-choisir/combien-pouvez-vous-emprunter, consulted on 21/05/2019

[67] https://www.wallonie.be/fr/actualites/prets-temperament-0-ecopack-et-renopack-fusionneront-en-2019, consulted on 20/05/2019

contribute people to invest if they expected it in a shorter horizon of time (larger house for the children for example).

A solution to those problems could result in a better information[68] of the Belgian citizens by dedicated marketing campaign on the returns and savings resulting from this investment or on the possibilities of 0% loan. For the tenant, a new agreement can be passed where both parties benefits from the solar panel installation (stable energy cost for the tenants and gains for the landlord with the CV revenues). Real estate transactions could incorporate the actualized gains of solar panels to incite households who change of house every 10 years to invest. It have to be done in consultation with the regional decision level, the companies and the citizens.

Impact on Planet

Investments in solar panel are a part of a sustainable economy which contribute to the energy transition. The profitability is higher than a stock exchange investment on the long term with lower risk. Most of the Value at Risk (VaR) and Expected Shortfall (ES) are positive. The returns are decorrelated with the financial exchanges. All those elements contribute to build a financial product that can play a role of diversification and by the same time improve sustainability rating of those products. Some could wait to have a bigger return, but it constitutes now a real alternative. One weak point that can be noted is the pollution induced at the end of life of the panels which need to be recycled to fully keep its benefits for the whole society.

Impact on Prosperity

The profitability of a solar panel is high, for the situation A1 (basis) the annual return is about 14% and for the Brussels region, it's 28%/year. Additionally, the risk is lower than an investment in an ETF. The returns are most of the time positive and don't suffer of losses when a financial crisis appears. The returns are mainly linked to the solar irradiance which don't varies across the time (excepted with the seasons) and electricity prices are sufficiently high to produce a minimum profitability in some regions the CV system provides even an additional cash-flows that almost multiplied the return by two.

Impact on Governance

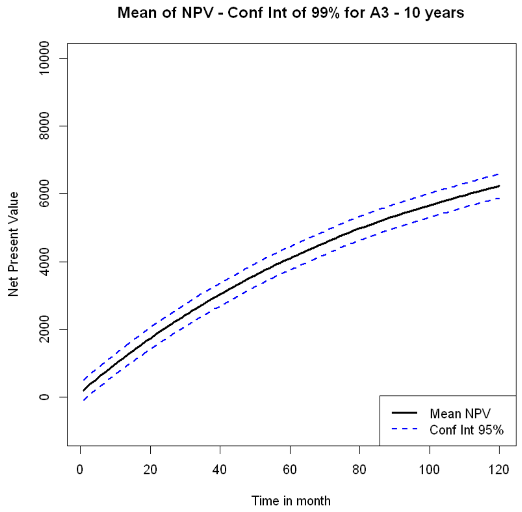
This model is useful to analyze the different policies applied in Belgium. The Brussels region is the most generous with the CV system, it considerably increases the profitability of the solar panel. On the other side Wallonia and Flanders use a Prosumer Tax which decreases the profitability but remains as high as 14% on average. The actual situation in 2019 in Wallonia is equivalent to the basis situation. With the Prosumer tax events, people hope to benefit from this tax regime for the whole life of the solar panel which is of course more profitable than the same situation with taxes. Even if it's applied, it will provide a sufficient level of profitability and people would not have to be scared of the tax amount.

Some calculus show that the maximal amount of Prosumer tax is:

- Wallonia is 290€ / kWc (price trend of 3% and start at 260€)
- Flanders about 270€/ kWc (price trend of 6% and start at 300€) or 181€/ kWc (price trend of 3% and start at 260€)

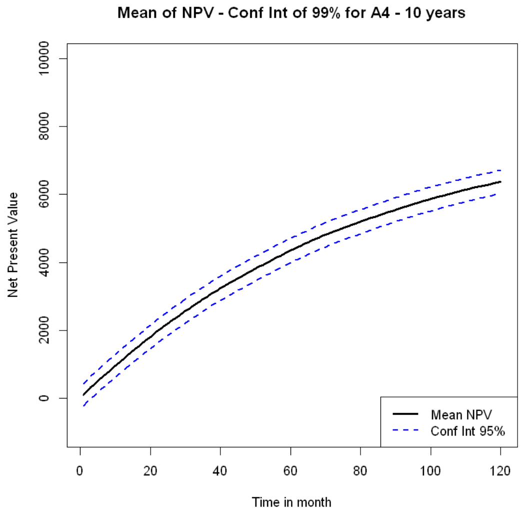
[68] Some systems exist already but need to be intensified.

Wallonia max Tax 290€ or 181€



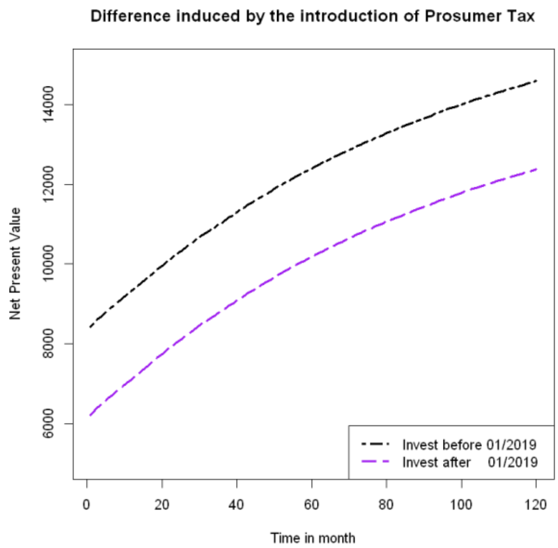
Mean of NPV at time 0:	201,22
Sd of NPV at time 0:	4.390,27
Real option value:	2.461,09
Optimal ratio:	1,07

Flanders max Tax 270€ or 181€
(depend of scenario)



Mean of NPV at time 0:	96,10
Sd of NPV at time 0:	4848,02
Real option value:	2704,80
Optimal ratio:	1,13

It's a useful tool for the governments of the different regions to avoid fixing a level of tax that can destroy all the investments in this technology. No analysis of the optimal level of CV has been conducted as without this additional help, the situation is equivalent to the scenario A1 which is largely profitable with a mean return of 19,8%. One precision must be made about the introduction of the Prosumer Tax in Wallonia. This series of events induces a high legal and political risk on the investments between 01/2019 and 04/2019. At the beginning of 2019, people were in the same condition than the scenario A1 (basis). With the announce of the introduction of a Prosumer Tax, people who invest just after the 07/2019 would be taxed as in the scenario A3. The difference of return can easily be compared with the simulation.



With the graph, a significant difference (2.210,53€) of mean in NPV at time 0 appears. It leads to a run to invest from the householders to avoid losing this amount. With the multiple events in the Walloon political world, this difference represents the cost of the uncertainty or the cost of a probable tax as it's equal to the actualized value of the probable tax outflows.

People need more stability and transparency with the political decisions that influence their investment decisions, for the solar panel it represents the largest risk as solar irradiance and electricity price are less volatile and more predictable that governments actions. It can be proved by the influence of the subsidy or tax regime: Brussels with a CV regime (return 28%/year) could switch to a Prosumer Tax situation (return 11%/year), the impact would be a run to invest before the effective change to gain an annual return twice larger than the situation after the tax introduction.

e) Next research opportunities

Next researches could be focused on a solar panel installation in another country of Europe to analyze the local legislation, subsidy or tax system that can be applied. Some countries with a high solar irradiance as Spain or Portugal could achieve similar profitability (or more) if the cost of energy is sufficiently high. About the LSM algorithm, it could be improved by using a better holding value function that forecast more smarter the option value or extended to other kinds of investment project. Finally, a genetic algorithm could test the level of an optimal ratio of investment among multiple scenarios.

f) Final thoughts

To summarize all the conclusions of the research, both optimal choices are presented with the situation in Belgium at the beginning of 2019:

- Wait as long as you want and get a higher profit when the investment is done
- Invest now and earn the profits during the life of the installation and reinvest it to earn more profit with the electricity price increase and investment cost decrease.

In the perspective of the energy transition and fight against the climate change, the second option could be better.

A comment about the different series of events:

- If the government announces a subsidy increase or a tax decrease: wait the effective change to earn more
- If the government announces a tax introduction, a subsidy decrease or a subsidy cancellation: invest now and hope that the change will never takes place even it's too optimistic

Example:
In 05/2019 when general elections happen in Belgium, the best choice could be:

- If a party announces a decrease of VAT on ecological investment:
  Wait the effective legislative change to buy the solar panel and avoid paying 21% instead of 6% VAT on the investment. It represents an immediate gain of 805,79€ (on an investment cost of 6500€ included of 21% VAT).
- If you think that the new majority in Wallonia will not introduce the Prosumer Tax:
  Invest now, you can gain an actualized amount of 2.210,53€
- If you think that the new majority in Wallonia will introduce the Prosumer Tax:
  Invest now and gain a cash-flows of 6 months not taxed (if invest in June)

Where invest in solar panel?
Flemish region has the highest prosumer tax, which is compensated with the highest electricity price, it gives an annual return of 12%.
Wallonia will have a probable prosumer tax and yield in this case an annual return of 11% as the electricity price are lower than in Flanders.
Brussels provides CV's which contribute to an annual return of 28%, the system is not supposed to stop as the granted number of CV is more stable and controlled than the situation in Wallonia in 2008. It's not impossible that the system will stop one day, in this case it will provide an annual return of 19,8%.

A bit of Belgian surrealism:

With the return in Brussels region, why don't convert a national monument[69] into a giant solar panel?

It could rapidly pay off.

[69] Image from: https://fr.wikipedia.org/wiki/Fichier:Belgium-6430B_-_Atomium_(14141441443).jpg, consulted on 21/05/2019

IV. Conclusion

The solar panels in Belgium suffer of a high uncertainty about the political decisions, especially in Wallonia. Data's reveal that investors wait to know the final regime of Tax or subsidies before investing. Real options are suited for this case. Historical rentability provided a return higher than 15% for a lower risk than a stock exchange market as most of the risk is based on the solar irradiance which is more predictable than the political decisions.

Based on a situation on the 01/01/2019, 3 models have been considered. The first advice is to wait but the results could not be correct as it doesn't consider the strong decrease of investment cost (-11%/year). The second model could not provide an analytical solution as no convergence is possible for this problem, again mainly due to the trend of investment cost and decorrelation of electricity price with the market. This missing solution could be interpreted as a signal to invest now. Waiting indefinitely produces higher profits. The third model was based on an LSM algorithm which produces a positive eternal option value. It means waiting the end of the option maturity (10 years) and the optimal solution is again waiting. To compare the results with a previous research [25], investment conditions in Belgium in 2019 are more favorable than the situation in China in 2016 as Belgian electricity prices are higher.

Analyzing the return of a solar panel installation with the EuroStoxx 50 and S&P500, the first one produces higher profits. Invest now constitutes also an alternative.

Distribution of Solar panel return		
Scenario	Mean of annual return	Sd of annual return
A1 - Basis	19,84%	0,005636
A2 - Brussels	30,45%	0,003291
A3 - Wallonia	12,68%	0,003741
A4 - Flanders	13,89%	0,005543

Distribution of Stock exchange return		
Indice	Mean of annual return	Sd of annual return
Eurostoxx 50	6,28%	0,222700
S&P 500	9,20%	0,158600

To summarize the answer to the question:

When it will be optimal to invest in solar panel in Belgium?

If the investor wants to rapidly earn profits from the solar panel and be protected of electricity price increase, invest now. If the investor wants a bigger profit than the results of the table above, wait but with the risk of legal changes that could deter the level of return.

This research could be extended to solar panel installations in countries with a higher solar irradiance rate or with different subsidy regimes, which should influence the total profitability and the investment decisions.

V. References

[1] R. A. Brealey, S. C. Myers and F. Allen, Principles of Corporate Finance, Mc Graw Hill Education, 2007.

[2] L. Tian, J. Pan, R. Du, W. Li, Z. Zhen and G. Qibing, "The valuation of photovoltaic power generation under carbon market linkage based on real options," *Applied Energy,* no. 201, 2017.

[3] A. G. &. J. v. d. Bergh, Real options analysis of investment in solar vs. wind energy: Diversification strategies under uncertain prices and costs, Renewable and Sustainable Energy Reviews 82, 2693–270, (2018).

[4] B. M. Lambrecht, "Real Option in Finance," *Journal of Banking and Finance,* vol. 81, pp. 161-171, 2017.

[5] J. C. Hull, Options, Futures and Other Derivatives, Pearson, 2017.

[6] D. Lamberton and B. Lapeyre, Introduction au calcul stochastique appliqué à la finance, Ellipses, 2012.

[7] F.-E. Racicot and R. Théoret, Finance computationnelle et gestion des risques, Presses de l'Université du Québec, 2006.

[8] A. K. Dixit and R. S. Pindyck, Investment under uncertainty, Princeton University Press, 1994.

[9] G. Stein, Chemical Reactions for the Utilization of Solar Energy, Plenum Press, 1977.

[10] CWAPE, "Rapport annuel spécifique 2017," 2017.

[11] J.-P. Hansen and J. Percebois, Energie, De Boeck, 2019.

[12] K.-T. Kim, D.-J. Jee and S.-J. Park, "Evaluation of R&D investments in wind power in Korea using real option," *Renewable and Sustainable Energy Reviews,* 2014 .

[13] G. Locatelli, D. Colette Invernizzi and M. Mancini, "Investment and risk appraisal in energy storage systems: A real option approach," *Energy,* vol. 104, 2016.

[14] E. Agliardia, E. Cattani and A. Ferrante, "Deep energy renovation strategies: A real option approach for add-ons in a social housing case study," *Energy and Buildings,* vol. 161, 2018.

[15] S.-E. Fleten, K. Linnerud, P. Molnar and M. Tandberg Nygaard, "Green electricity investment timing in practice: Real options or net present value ?," *Energy,* vol. 116, 2016.

[16] F. A. Longstaff and E. S. Schwartz, "Valuing American Options by Simulation: A Simple Least-Squares Approach," *The Reiieib of Firiancial Studies,* vol. 14, 2011.

[17] L. Tian, J. Pan, R. Du, L. Wenchao, Z. Zhen and G. Qibing, "The valuation of photovoltaic power generation under carbon market linkage based on real options," *Applied Energy,* vol. 201, 2017.

[18] L. Zhu and Y. Fan, "A real options-based CCS investment evaluation model: Case study of China's power generation sector," *Applied Energy,* vol. 88, 2011.

[19] L. Zhu, "A simulation based real options approach for the investment evaluation of nuclear power," *Computers & Industrial Engineering,* vol. 63, 2012.

[20] E. Martinez-Cesena and J. Mutale, "Application of an advanced real options approach for renewable energy generation projects planning," *Renewable and Sustainable Energy Reviews,* vol. 15, 2011.

[21] C. Jeon and J. Shin, "Long-term renewable energy technology valuation using system dynamics and Monte Carlo simulation: Photovoltaic technology case," *Energy,* vol. 66, 2014.

[22] M. Zhang, P. Zhou and D. Zhou, "A real options model for renewable energy investment with application to solar photovoltaic power generation in China," *Energy Economics,* vol. 59, 2016.

[23] R. McDonald and D. Siegel, "The Value of Waiting to Invest," *The Quartely Journal of Economics,* vol. 101, pp. 707-728, 1986.

[24] CREG, "Etude relative aux composantes des prix de l'électricité et du gaz naturel," 2019.

[25] M. Zhang, P. Zhou and D. Zhou, "A real options model for renewable energy investment with applicationto solar photovoltaic power generation in China," *Energy Economics,* vol. 59, 2016.

[26] D. G. Brooks, The Sampling Distribution and Central Limit Theorem, CreateSpace Independent Publishing Platform, 2012.

VI. Figure table

Annexes

Annexes

Created by Valepin Rémi – R Code of the LSM model

Master Thesis: Least-Square Monte Carlo applied to a solar panel investment in Belgium, starting datum 01/01/2019

The function

The function generates a data-frame with the results where Real Option Value of the process and Optimal ratio of investment is summarized with the function "Test_summary"

The function generates Inf error messages, it's normal for the paths without activation of the option.

In [2]:

```
# Activate twice

library(FinancialMath)

library(zoo)

library(xts)

library(PerformanceAnalytics)
```

In [3]:

```
LSM_solar_belgium <- function(Year = 10, Period = 12, N_sim = 10, Life_y_installation = 20,

                        Elec_price_start = 260, Elec_trend_yearly = 0.03, Interest_rate_yearly = 0.03) {

  ### Initialize values
  Core_data         <- array(0,dim=c((Year+Life_y_installation)*Period,87,N_sim)) # 01
  #E              <- array(0,dim=c(n*T,1,sim)) # 02
  #Q              <- array(0,dim=c(n*T,1,sim)) # 03
  #CV             <- array(0,dim=c(n*T,1,sim)) # 04
  #PRO_WL          <- array(0,dim=c(n*T,1,sim)) # 05
  #PRO_FL         <- array(0,dim=c(n*T,1,sim)) # 06
  #I             <- array(0,dim=c(n*T,1,sim)) # 07
  #Flow_A1        <- array(0,dim=c(n*T,1,sim)) # 08
  #Flow_A2        <- array(0,dim=c(n*T,1,sim)) # 09
  #Flow_A3        <- array(0,dim=c(n*T,1,sim)) # 10
  #Flow_A4        <- array(0,dim=c(n*T,1,sim)) # 11
  #Present_value_A1 <- array(0,dim=c(n*T,1,sim)) # 12
  #Present_value_A2 <- array(0,dim=c(n*T,1,sim)) # 13
  #Present_value_A3 <- array(0,dim=c(n*T,1,sim)) # 14
  #Present_value_A4 <- array(0,dim=c(n*T,1,sim)) # 15
  #Present_value_A5 <- array(0,dim=c(n*T,1,sim)) # 16
  #Horizon_t       <- array(0,dim=c(n*T,1,sim)) # 17
  #PV_I_A1        <- array(0,dim=c(n*T,1,sim)) # 18
  #PV_I_A2        <- array(0,dim=c(n*T,1,sim)) # 19
  #PV_I_A3        <- array(0,dim=c(n*T,1,sim)) # 20
  #PV_I_A4        <- array(0,dim=c(n*T,1,sim)) # 21
  #PV_I_A5        <- array(0,dim=c(n*T,1,sim)) # 22
  #PV_I_Best_A1     <- array(0,dim=c(n*T,1,sim)) # 23
  #PV_I_Best_A2     <- array(0,dim=c(n*T,1,sim)) # 24
  #PV_I_Best_A3     <- array(0,dim=c(n*T,1,sim)) # 25
  #PV_I_Best_A4     <- array(0,dim=c(n*T,1,sim)) # 26
  #PV_I_Best_A5     <- array(0,dim=c(n*T,1,sim)) # 27
  #PV_I_A1_ratio    <- array(0,dim=c(n*T,1,sim)) # 28
  #PV_I_A2_ratio    <- array(0,dim=c(n*T,1,sim)) # 29
```

```
#PV_I_A3_ratio    <- array(0,dim=c(n*T,1,sim)) # 30
#PV_I_A4_ratio    <- array(0,dim=c(n*T,1,sim)) # 31
#PV_I_A5_ratio    <- array(0,dim=c(n*T,1,sim)) # 32
#Actu_PV_best_A1  <- array(0,dim=c(n*T,1,sim)) # 33
#Actu_PV_best_A2  <- array(0,dim=c(n*T,1,sim)) # 34
#Actu_PV_best_A3  <- array(0,dim=c(n*T,1,sim)) # 35
#Actu_PV_best_A4  <- array(0,dim=c(n*T,1,sim)) # 36
#Actu_PV_best_A5  <- array(0,dim=c(n*T,1,sim)) # 37
#Pay_off_A1       <- array(0,dim=c(n*T,1,sim)) # 38
#Pay_off_A2       <- array(0,dim=c(n*T,1,sim)) # 39
#Pay_off_A3       <- array(0,dim=c(n*T,1,sim)) # 40
#Pay_off_A4       <- array(0,dim=c(n*T,1,sim)) # 41
#Pay_off_A5       <- array(0,dim=c(n*T,1,sim)) # 42
#Conti_value_A1   <- array(0,dim=c(n*T,1,sim)) # 43
#Conti_value_A2   <- array(0,dim=c(n*T,1,sim)) # 44
#Conti_value_A3   <- array(0,dim=c(n*T,1,sim)) # 45
#Conti_value_A4   <- array(0,dim=c(n*T,1,sim)) # 46
#Conti_value_A5   <- array(0,dim=c(n*T,1,sim)) # 47
#Dummy_info_A1    <- array(0,dim=c(n*T,1,sim)) # 48
#Dummy_info_A2    <- array(0,dim=c(n*T,1,sim)) # 49
#Dummy_info_A3    <- array(0,dim=c(n*T,1,sim)) # 50
#Dummy_info_A4    <- array(0,dim=c(n*T,1,sim)) # 51
#Dummy_info_A5    <- array(0,dim=c(n*T,1,sim)) # 52
#Dummy_min_A1     <- array(0,dim=c(n*T,1,sim)) # 53
#Dummy_min_A2     <- array(0,dim=c(n*T,1,sim)) # 54
#Dummy_min_A3     <- array(0,dim=c(n*T,1,sim)) # 55
#Dummy_min_A4     <- array(0,dim=c(n*T,1,sim)) # 56
#Dummy_min_A5     <- array(0,dim=c(n*T,1,sim)) # 57
#Best_payoff_A1   <- array(0,dim=c(n*T,1,sim)) # 58
#Best_payoff_A2   <- array(0,dim=c(n*T,1,sim)) # 59
#Best_payoff_A3   <- array(0,dim=c(n*T,1,sim)) # 60
#Best_payoff_A4   <- array(0,dim=c(n*T,1,sim)) # 61
#Best_payoff_A5   <- array(0,dim=c(n*T,1,sim)) # 62
#ROV_brut_A1      <- array(0,dim=c(n*T,1,sim)) # 63
#ROV_brut_A2      <- array(0,dim=c(n*T,1,sim)) # 64
#ROV_brut_A3      <- array(0,dim=c(n*T,1,sim)) # 65
#ROV_brut_A4      <- array(0,dim=c(n*T,1,sim)) # 66
#ROV_brut_A5      <- array(0,dim=c(n*T,1,sim)) # 67
#Rolling_mean_A1  <- array(0,dim=c(n*T,1,sim)) # 68
#Rolling_mean_A2  <- array(0,dim=c(n*T,1,sim)) # 69
#Rolling_mean_A3  <- array(0,dim=c(n*T,1,sim)) # 70
#Rolling_mean_A4  <- array(0,dim=c(n*T,1,sim)) # 71
#Rolling_mean_A5  <- array(0,dim=c(n*T,1,sim)) # 72
#ROV_brut_rat_A1  <- array(0,dim=c(n*T,1,sim)) # 73
#ROV_brut_rat_A2  <- array(0,dim=c(n*T,1,sim)) # 74
#ROV_brut_rat_A3  <- array(0,dim=c(n*T,1,sim)) # 75
#ROV_brut_rat_A4  <- array(0,dim=c(n*T,1,sim)) # 76
#ROV_brut_rat_A5  <- array(0,dim=c(n*T,1,sim)) # 77
#ROV_ratio_A1     <- array(0,dim=c(n*T,1,sim)) # 78
#ROV_ratio_A2     <- array(0,dim=c(n*T,1,sim)) # 79
#ROV_ratio_A3     <- array(0,dim=c(n*T,1,sim)) # 80
#ROV_ratio_A4     <- array(0,dim=c(n*T,1,sim)) # 81
#ROV_ratio_A5     <- array(0,dim=c(n*T,1,sim)) # 82
#ROV_ratio_rm_A1  <- array(0,dim=c(n*T,1,sim)) # 83
#ROV_ratio_rm_A2  <- array(0,dim=c(n*T,1,sim)) # 84
#ROV_ratio_rm_A3  <- array(0,dim=c(n*T,1,sim)) # 85
#ROV_ratio_rm_A4  <- array(0,dim=c(n*T,1,sim)) # 86
#ROV_ratio_rm_A5  <- array(0,dim=c(n*T,1,sim)) # 87
```

```
dt          <- 1/Period
Power       <- 3 # expressed in kWc
CV_granting  <- 3 # by 1000 kWh

E_start      <- Elec_price_start
Q_start      <- 104.96
CV_start     <- 96
PRO_WL_start <- 78 *Power*(1-0.3736) #78
PRO_FL_start <- 110 *Power #110
I_start      <- 6500

E_alpha      <- Elec_trend_yearly
Q_alpha      <- 0.005
CV_alpha     <- 0.003
PRO_WL_alpha <- 0.00
PRO_FL_alpha <- 0.00
I_alpha      <- -0.115 #-0.115

E_sd         <- 0.2
Q_sd         <- 126.27
Q_mean       <- 104.96
Q_return     <- 3.1033
CV_sd        <- 0.03
PRO_WL_sd    <- 0.0
PRO_FL_sd    <- 0.0
I_sd         <- 0.075



Maintenance  <- 0.0075*I_start
Loss_factor  <- 0.005
Inverter_cost<- 250

R_int        <- (1+Interest_rate_yearly)^(dt)-1
R_adjust     <- 0.00
R_M_int      <- 0.003928+ R_adjust
R_M_sd       <- 0.018

Corr_E_M     <- -0.04
Corr_I_M     <-  0.32
Corr_CV_M    <-  0.175

E_crol       <- R_int + ((R_M_int-R_int)/R_M_sd)*E_sd *Corr_E_M  - E_alpha
CV_crol      <- R_int + ((R_M_int-R_int)/R_M_sd)*CV_sd*Corr_CV_M - CV_alpha
I_crol       <- R_int + ((R_M_int-R_int)/R_M_sd)*I_sd *Corr_I_M  - I_alpha

### First period set
Core_data[1,1,] <- 1
Core_data[1,2,] <- E_start
Core_data[1,3,] <- Q_start
Core_data[1,4,] <- CV_start
Core_data[1,5,] <- PRO_WL_start
Core_data[1,6,] <- PRO_FL_start
Core_data[1,7,] <- I_start

### Set valuation's rules

# Reminder
# Flow[1] <- P[1]*Q[1] - A1*(PRO[1]*Q[1]) - A2*PRO_FL - Maintenance + A3*Qualiwatt +
A4*(CV[1]*Q[1]) + A5*Other_subsidy
```

```
  # A1
  Core_data[1,8,]  <- Core_data[1,2,]*Core_data[1,3,]*3*0.86/1000 - Maintenance*dt
  # A2
  Core_data[1,9,]  <- Core_data[1,2,]*Core_data[1,3,]*3*0.86/1000 - Maintenance*dt + 
3*Core_data[1,4,]*Core_data[1,3,]*3*0.86/1000
  # A3
  Core_data[1,10,] <- Core_data[1,2,]*Core_data[1,3,]*3*0.86/1000 - Maintenance*dt - 
Core_data[1,5,]*dt
  # A4
  Core_data[1,11,] <- Core_data[1,2,]*Core_data[1,3,]*3*0.86/1000 - Maintenance*dt - 
Core_data[1,6,]*dt
  # Time max
  Core_data[1,17,] <- Life_y_installation*Period

  for (j in seq(from= 1, to = N_sim, by = 1) ) {

  ### Set Paths of the variables

    for (i in seq(from= 2, to = ((Year+Life_y_installation)*Period), by = 1) ) {

      # Time
      Core_data[i,1,j]  = Core_data[i-1,1,j] + 1
      # BM E
      #Core_data[i,2,j]  = Core_data[i-1,2,j] * exp(((E_alpha-((E_sd)^(2))/2)*dt)+E_sd*sqrt(dt)*rnorm(1,0))
      Core_data[i,2,j]  = Core_data[i-1,2,j] + ((Core_data[i-1,2,j] *E_alpha  *dt)) + (Core_data[i-1,2,j] 
*E_sd  *sqrt(dt)*rnorm(1,0))
      Core_data[i,2,j]  = min(max(Core_data[i,2,j],180),700)
      # BM Q
      Core_data[i,3,j]  = Core_data[i-1,3,j] + ((Q_mean-Core_data[i-1,3,j])*Q_return*dt) + (Q_sd 
*sqrt(dt)*rnorm(1,0))
      Core_data[i,3,j]  = min(max(Core_data[i,3,j],20),200)
      # BM CV
      Core_data[i,4,j]  = Core_data[i-1,4,j] + ((Core_data[i-1,4,j] *CV_alpha *dt)) + (Core_data[i-1,4,j] 
*CV_sd *sqrt(dt)*rnorm(1,0))
      #Core_data[i,4,j]  = Core_data[i-1,4,j] * exp(((CV_alpha-
((CV_sd)^(2))/2)*dt)+CV_sd*sqrt(dt)*rnorm(1,0))
      Core_data[i,4,j]  = min(max(65, Core_data[i,4,j]),100)
      # BM PRO_WL
      Core_data[i,5,j]  = Core_data[i-1,5,j] + ((Core_data[i-1,5,j] *PRO_WL_alpha*dt))
      # BM PRO_FL
      Core_data[i,6,j]  = Core_data[i-1,6,j] + ((Core_data[i-1,6,j] *PRO_FL_alpha*dt))
      # BM I
      Core_data[i,7,j]  = Core_data[i-1,7,j] + ((Core_data[i-1,7,j]*I_alpha  *dt)) + (Core_data[i-1,7,j]*I_sd 
*sqrt(dt)*rnorm(1,0))
      #Core_data[i,7,j]  = Core_data[i-1,7,j] * exp(((I_alpha-((I_sd)^(2))/2)*dt)+I_sd*sqrt(dt)*rnorm(1,0))
      Core_data[i,7,j]  = min(max(Core_data[i,7,j],500),7000)

      # Set A1
      Core_data[i,8,j]  = Core_data[i,2,j]*Core_data[i,3,j]*3*0.86/1000*(1-
Loss_factor*Core_data[i,1,j]*dt) - Maintenance*dt
      #Core_data[i,8,j]  = (Core_data[i,2,j]*(Core_data[i,3,j])*(1-Loss_factor*Core_data[i,1,j]*dt)) - 
Maintenance*dt

      # Set A2
      Core_data[i,9,j]  = Core_data[i,2,j]*Core_data[i,3,j]*3*0.86/1000*(1-
Loss_factor*Core_data[i,1,j]*dt) - Maintenance*dt + 3*Core_data[i,4,j]*Core_data[i,3,j]*3*0.86/1000
      # Set A3
      Core_data[i,10,j] = Core_data[i,2,j]*Core_data[i,3,j]*3*0.86/1000*(1-
Loss_factor*Core_data[i,1,j]*dt) - Maintenance*dt - Core_data[i,5,j]*dt
      # Set A4
```

```
    Core_data[i,11,j] = Core_data[i,2,j]*Core_data[i,3,j]*3*0.86/1000*(1-
Loss_factor*Core_data[i,1,j]*dt) - Maintenance*dt - Core_data[i,6,j]*dt

    # Set A1
    Core_data[i,17,j] = min(Core_data[i,1,j]+(Life_y_installation*Period),
((Year+Life_y_installation)*Period))
   }

  ### Set Present Value of Flow
   for (i in seq(from= 1, to = (Year+Life_y_installation)*Period, by = 1) ) {
     # PV A1
     Core_data[i,12,j] = NPV(0,Core_data[i:Core_data[i,17,j],8,j],seq(from=1, to=
length(Core_data[i:Core_data[i,17,j],8,j]),  by = 1), R_int)-Inverter_cost*1/(1+R_int)^(120)
     # PV A2
     Core_data[i,13,j] = NPV(0,Core_data[i:Core_data[i,17,j],9,j],seq(from=1, to=
length(Core_data[i:Core_data[i,17,j],9,j]),  by = 1), R_int)-Inverter_cost*1/(1+R_int)^(120)
     # PV A3
     Core_data[i,14,j] = NPV(0,Core_data[i:Core_data[i,17,j],10,j],seq(from=1, to=
length(Core_data[i:Core_data[i,17,j],10,j]),  by = 1), R_int)-Inverter_cost*1/(1+R_int)^(120)
     # PV A4
     Core_data[i,15,j] = NPV(0,Core_data[i:Core_data[i,17,j],11,j],seq(from=1, to=
length(Core_data[i:Core_data[i,17,j],11,j]), by = 1), R_int)-Inverter_cost*1/(1+R_int)^(120)

     # NPV A1
     Core_data[i,18,j] = Core_data[i,12,j] - Core_data[i,7,j]
     # NPV A2
     Core_data[i,19,j] = Core_data[i,13,j] - Core_data[i,7,j]
     # NPV A3
     Core_data[i,20,j] = Core_data[i,14,j] - Core_data[i,7,j]
     # NPV A4
     Core_data[i,21,j] = Core_data[i,15,j] - Core_data[i,7,j]

     # Termination value A1
     Core_data[i,23,j] = max(Core_data[i,18,j], 0)
     # Termination value A2
     Core_data[i,24,j] = max(Core_data[i,19,j], 0)
     # Termination value A3
     Core_data[i,25,j] = max(Core_data[i,20,j], 0)
     # Termination value A4
     Core_data[i,26,j] = max(Core_data[i,21,j], 0)

     # Invest ratio A1
     Core_data[i,28,j] = Core_data[i,12,j]/Core_data[i,7,j]
     # Invest ratio A2
     Core_data[i,29,j] = Core_data[i,13,j]/Core_data[i,7,j]
     # Invest ratio A3
     Core_data[i,30,j] = Core_data[i,14,j]/Core_data[i,7,j]
     # Invest ratio A4
     Core_data[i,31,j] = Core_data[i,15,j]/Core_data[i,7,j]

  }

    # Termination value A1
     Core_data[121,23,j] = 0
     # Termination value A2
     Core_data[121,24,j] = 0
     # Termination value A3
     Core_data[121,25,j] = 0
     # Termination value A4
     Core_data[121,26,j] = 0
 }
```

```
### Set Roll-back value
for (i in seq(from= (((Year+Life_y_installation)*Period)), to = 1, by = -1) ) {

    for (j in seq(from= 1, to = N_sim, by = 1) ) {
        Core_data[i,33,j] = Core_data[i,23,j] * 1/(1+R_int)
        Core_data[i,34,j] = Core_data[i,24,j] * 1/(1+R_int)
        Core_data[i,35,j] = Core_data[i,25,j] * 1/(1+R_int)
        Core_data[i,36,j] = Core_data[i,26,j] * 1/(1+R_int)
    }
}

### Set again terminal payoff of the option
for (j in seq(from= 1, to = N_sim, by = 1) ) {
     Core_data[((Year)*Period),38,j] = Core_data[((Year)*Period),23,j]
     Core_data[((Year)*Period),39,j] = Core_data[((Year)*Period),24,j]
     Core_data[((Year)*Period),40,j] = Core_data[((Year)*Period),25,j]
     Core_data[((Year)*Period),41,j] = Core_data[((Year)*Period),26,j]
}

### Compute expectations through linear regression
for (i in seq(from= ((Year)*Period), to = 2, by = -1) ) {

    #V_E  <- Core_data[i,2,]
    #V_Q  <- Core_data[i,3,]
    #V_CV <- Core_data[i,4,]
    #V_I  <- Core_data[i,7,]

    #fit_A1 =
lm(Core_data[i,33,]~V_E+I(V_E^2)+V_Q+I(V_Q^2)+V_I+I(V_I^2)+I(V_E*V_Q)+I(V_E*V_I)+I(V_Q*V_I))
    #fit_A2 =
lm(Core_data[i,33,]~V_E+I(V_E^2)+V_Q+I(V_Q^2)+V_I+I(V_I^2)+I(V_E*V_Q)+I(V_E*V_I)+I(V_Q*V_I)+V
_CV+I(V_CV^2)+I(V_E*V_CV)+I(V_Q*V_CV)+I(V_I*V_CV))
    #fit_A3 =
lm(Core_data[i,33,]~V_E+I(V_E^2)+V_Q+I(V_Q^2)+V_I+I(V_I^2)+I(V_E*V_Q)+I(V_E*V_I)+I(V_Q*V_I)+C
ore_data[i,5,j])
    #fit_A4 =
lm(Core_data[i,33,]~V_E+I(V_E^2)+V_Q+I(V_Q^2)+V_I+I(V_I^2)+I(V_E*V_Q)+I(V_E*V_I)+I(V_Q*V_I)+C
ore_data[i,6,j])

   fit_A1 =
lm(Core_data[i,33,]~Core_data[i,2,]+I(Core_data[i,2,]^2)+Core_data[i,3,]+I(Core_data[i,3,]^2)+Core_dat
a[i,7,]+I(Core_data[i,7,]^2)+I(Core_data[i,2,]*Core_data[i,3,])+I(Core_data[i,2,]*Core_data[i,7,])+I(Core_
data[i,3,]*Core_data[i,7,]))
   fit_A2 =
lm(Core_data[i,34,]~Core_data[i,2,]+I(Core_data[i,2,]^2)+Core_data[i,3,]+I(Core_data[i,3,]^2)+Core_dat
a[i,7,]+I(Core_data[i,7,]^2)+I(Core_data[i,2,]*Core_data[i,3,])+I(Core_data[i,2,]*Core_data[i,7,])+I(Core_
data[i,3,]*Core_data[i,7,])+Core_data[i,4,]+I(Core_data[i,4,]^2)+I(Core_data[i,2,]*Core_data[i,4,])+I(Core
_data[i,3,]*Core_data[i,4,])+I(Core_data[i,7,]*Core_data[i,4,]))
   fit_A3 =
lm(Core_data[i,35,]~Core_data[i,2,]+I(Core_data[i,2,]^2)+Core_data[i,3,]+I(Core_data[i,3,]^2)+Core_dat
a[i,7,]+I(Core_data[i,7,]^2)+I(Core_data[i,2,]*Core_data[i,3,])+I(Core_data[i,2,]*Core_data[i,7,])+I(Core_
data[i,3,]*Core_data[i,7,]))
   fit_A4 =
lm(Core_data[i,36,]~Core_data[i,2,]+I(Core_data[i,2,]^2)+Core_data[i,3,]+I(Core_data[i,3,]^2)+Core_dat
a[i,7,]+I(Core_data[i,7,]^2)+I(Core_data[i,2,]*Core_data[i,3,])+I(Core_data[i,2,]*Core_data[i,7,])+I(Core_
data[i,3,]*Core_data[i,7,]))


    for (j in seq(from= 1, to = N_sim, by = 1) ) {

        fit_value_A1 <- as.numeric(
```

```
                         fit_A1$coefficients[1]*1
                        +fit_A1$coefficients[2]*Core_data[i,2,j]
                        +fit_A1$coefficients[3]*Core_data[i,2,j]^(2)
                        +fit_A1$coefficients[4]*Core_data[i,3,j]
                        +fit_A1$coefficients[5]*Core_data[i,3,j]^(2)
                        +fit_A1$coefficients[6]*Core_data[i,7,j]
                        +fit_A1$coefficients[7]*Core_data[i,7,j]^(2)
                        +fit_A1$coefficients[8]*Core_data[i,2,j]*Core_data[i,3,j]
                        +fit_A1$coefficients[9]*Core_data[i,2,j]*Core_data[i,7,j]
                        +fit_A1$coefficients[10]*Core_data[i,3,j]*Core_data[i,7,j]
                       )

    fit_value_A2 <- as.numeric(
                         fit_A2$coefficients[1]*1
                        +fit_A2$coefficients[2]*Core_data[i,2,j]
                        +fit_A2$coefficients[3]*Core_data[i,2,j]^(2)
                        +fit_A2$coefficients[4]*Core_data[i,3,j]
                        +fit_A2$coefficients[5]*Core_data[i,3,j]^(2)
                        +fit_A2$coefficients[6]*Core_data[i,7,j]
                        +fit_A2$coefficients[7]*Core_data[i,7,j]^(2)
                        +fit_A2$coefficients[8]*Core_data[i,2,j]*Core_data[i,3,j]
                        +fit_A2$coefficients[9]*Core_data[i,2,j]*Core_data[i,7,j]
                        +fit_A2$coefficients[10]*Core_data[i,3,j]*Core_data[i,7,j]
                        +fit_A2$coefficients[11]*Core_data[i,4,j]
                        +fit_A2$coefficients[12]*Core_data[i,4,j]^(2)
                        +fit_A2$coefficients[13]*Core_data[i,4,j]*Core_data[i,2,j]
                        +fit_A2$coefficients[14]*Core_data[i,4,j]*Core_data[i,3,j]
                        +fit_A2$coefficients[15]*Core_data[i,4,j]*Core_data[i,7,j]
                       )

    fit_value_A3 <- as.numeric(
                         fit_A3$coefficients[1]*1
                        +fit_A3$coefficients[2]*Core_data[i,2,j]
                        +fit_A3$coefficients[3]*Core_data[i,2,j]^(2)
                        +fit_A3$coefficients[4]*Core_data[i,3,j]
                        +fit_A3$coefficients[5]*Core_data[i,3,j]^(2)
                        +fit_A3$coefficients[6]*Core_data[i,7,j]
                        +fit_A3$coefficients[7]*Core_data[i,7,j]^(2)
                        +fit_A3$coefficients[8]*Core_data[i,2,j]*Core_data[i,3,j]
                        +fit_A3$coefficients[9]*Core_data[i,2,j]*Core_data[i,7,j]
                        +fit_A3$coefficients[10]*Core_data[i,3,j]*Core_data[i,7,j]
                       )

    fit_value_A4 <- as.numeric(
                         fit_A4$coefficients[1]*1
                        +fit_A4$coefficients[2]*Core_data[i,2,j]
                        +fit_A4$coefficients[3]*Core_data[i,2,j]^(2)
                        +fit_A4$coefficients[4]*Core_data[i,3,j]
                        +fit_A4$coefficients[5]*Core_data[i,3,j]^(2)
                        +fit_A4$coefficients[6]*Core_data[i,7,j]
                        +fit_A4$coefficients[7]*Core_data[i,7,j]^(2)
                        +fit_A4$coefficients[8]*Core_data[i,2,j]*Core_data[i,3,j]
                        +fit_A4$coefficients[9]*Core_data[i,2,j]*Core_data[i,7,j]
                        +fit_A4$coefficients[10]*Core_data[i,3,j]*Core_data[i,7,j]
                       )


### Set Continuation value at each instant t
    Core_data[i,43,j] = max(as.numeric(fit_value_A1),0)
    Core_data[i,44,j] = max(as.numeric(fit_value_A2),0)
    Core_data[i,45,j] = max(as.numeric(fit_value_A3),0)
```

```
          Core_data[i,46,j] = max(as.numeric(fit_value_A4),0)

    ### Choose best payoff between activation or continuation value
          Core_data[i,38,j] = max(Core_data[i,23,j], Core_data[i,43,j])
          Core_data[i,39,j] = max(Core_data[i,24,j], Core_data[i,44,j])
          Core_data[i,40,j] = max(Core_data[i,25,j], Core_data[i,45,j])
          Core_data[i,41,j] = max(Core_data[i,26,j], Core_data[i,46,j])

    ### Dummy variable for this choice
          # Dummy A1
          if (Core_data[i,23,j] >= Core_data[i,43,j]) {
            Core_data[i,48,j] = 1
          }
          else {
            Core_data[i,48,j] = 0
          }
          # Dummy A2
          if (Core_data[i,24,j] >= Core_data[i,44,j]) {
            Core_data[i,49,j] = 1
          }
          else {
            Core_data[i,49,j] = 0
          }
          # Dummy A3
          if (Core_data[i,25,j] >= Core_data[i,45,j]) {
            Core_data[i,50,j] = 1
          }
          else {
            Core_data[i,50,j] = 0
          }
          # Dummy A4
          if (Core_data[i,26,j] >= Core_data[i,46,j]) {
            Core_data[i,51,j] = 1
          }
          else {
            Core_data[i,51,j] = 0
          }
      }
    }

    ### Determine best time and payoff

    for (j in seq(from= 1, to = N_sim, by = 1) ) {
        Core_data[1,53,j] = if (is.infinite(min(which(Core_data[,48,j]==1)))) { 0 } else {
min(which(Core_data[,48,j]==1)) }
        Core_data[1,54,j] = if (is.infinite(min(which(Core_data[,49,j]==1)))) { 0 } else {
min(which(Core_data[,48,j]==1)) }
        Core_data[1,55,j] = if (is.infinite(min(which(Core_data[,50,j]==1)))) { 0 } else {
min(which(Core_data[,48,j]==1)) }
        Core_data[1,56,j] = if (is.infinite(min(which(Core_data[,51,j]==1)))) { 0 } else {
min(which(Core_data[,48,j]==1)) }
    }

    for (j in seq(from= 1, to = N_sim, by = 1) ) {
        Core_data[1,58,j] = if (Core_data[1,53,j]==0) { 0 } else {
Core_data[Core_data[1,53,j],38,j]*1/(1+R_int)^(Core_data[1,53,j]) }
        Core_data[1,59,j] = if (Core_data[1,54,j]==0) { 0 } else {
Core_data[Core_data[1,54,j],39,j]*1/(1+R_int)^(Core_data[1,54,j]) }
        Core_data[1,60,j] = if (Core_data[1,55,j]==0) { 0 } else {
Core_data[Core_data[1,55,j],40,j]*1/(1+R_int)^(Core_data[1,55,j]) }
```

```
        Core_data[1,61,j] = if (Core_data[1,56,j]==0) { 0 } else {
Core_data[Core_data[1,56,j],41,j]*1/(1+R_int)^(Core_data[1,56,j]) }

        Core_data[1,73,j] = if (Core_data[1,53,j]==0) { 0 } else { Core_data[Core_data[1,53,j],28,j] }
        Core_data[1,74,j] = if (Core_data[1,54,j]==0) { 0 } else { Core_data[Core_data[1,54,j],29,j] }
        Core_data[1,75,j] = if (Core_data[1,55,j]==0) { 0 } else { Core_data[Core_data[1,55,j],30,j] }
        Core_data[1,76,j] = if (Core_data[1,56,j]==0) { 0 } else { Core_data[Core_data[1,56,j],31,j] }
    }

    ### Determine Real Option Value
    # ROV A1
    Core_data[1,63,1] = mean(Core_data[1,58,], na.rm = TRUE)
    # ROV A2
    Core_data[1,64,1] = mean(Core_data[1,59,], na.rm = TRUE)
    # ROV A3
    Core_data[1,65,1] = mean(Core_data[1,60,], na.rm = TRUE)
    # ROV A4
    Core_data[1,66,1] = mean(Core_data[1,61,], na.rm = TRUE)
    # Optimal ratio A1
    Core_data[1,78,1] = mean(Core_data[1,73,], na.rm = TRUE)
    # Optimal ratio A2
    Core_data[1,79,1] = mean(Core_data[1,74,], na.rm = TRUE)
    # Optimal ratio A3
    Core_data[1,80,1] = mean(Core_data[1,75,], na.rm = TRUE)
    # Optimal ratio A4
    Core_data[1,81,1] = mean(Core_data[1,76,], na.rm = TRUE)


    ### Graph data to convergence
    for (j in seq(from= 1, to = N_sim, by = 1) ) {
        # ROV A1 convergence
        Core_data[1,68,j] = mean(Core_data[1,58,1:j], na.rm = TRUE)
        # ROV A2 convergence
        Core_data[1,69,j] = mean(Core_data[1,59,1:j], na.rm = TRUE)
        # ROV A3 convergence
        Core_data[1,70,j] = mean(Core_data[1,60,1:j], na.rm = TRUE)
        # ROV A4 convergence
        Core_data[1,71,j] = mean(Core_data[1,61,1:j], na.rm = TRUE)

        # Optimal ratio A1 convergence
        Core_data[1,83,j] = mean(Core_data[1,73,1:j], na.rm = TRUE)
        # Optimal ratio A2 convergence
        Core_data[1,84,j] = mean(Core_data[1,74,1:j], na.rm = TRUE)
        # Optimal ratio A3 convergence
        Core_data[1,85,j] = mean(Core_data[1,75,1:j], na.rm = TRUE)
        # Optimal ratio A4 convergence
        Core_data[1,86,j] = mean(Core_data[1,76,1:j], na.rm = TRUE)

    }

    return(Core_data)
}
```

In [4]:

```
Test_summary <- function(x) {
    table_summary <- data.frame(c(x[1,63,1],x[1,64,1],x[1,65,1],x[1,66,1]),
                                c(x[1,78,1],x[1,79,1],x[1,80,1],x[1,81,1]))

    colnames(table_summary) <- c("Real Option Value","Optimal ratio")
    rownames(table_summary) <- c("A1","A2","A3","A4")

    return(table_summary) }
```